Electrostatics

UNIT 1 Solutions

1.1 Electrostatic Fields

Q1. Discuss the term "Electric field due to a static charge configuration".

Feb.-08, Set-3, Q1(a) M[6]

Answer :

The space or region around an electric charge in which the effect of charge is felt is termed as Electric field or Electrostatic field. A static electric field may be due to a positive charge or a negative charge. The presence of electric field or the absence of electric field in a region is confirmed by bringing the test charge into a region. If a test charge experiences a force, then it means that the field is present. If a small positive charge is brought into the field of a positive charge, it experiences a repulsive force and if the field is of negative charge and the test charge is positive, it experiences the force of attraction.

Electric Field Due to a Static Charge Configuration

"Electric field is defined as the force per unit charge provided the charge being as small as possible".

If 'F' is the force acting on the charge 'q' then electric field intensity is given by,

$$E = \underset{q \to 0}{Lt}\left(\frac{F}{q}\right) \text{N/C}$$

The test charge 'q' must be so small that it should not disturb the properties of the field, i.e., it should exert negligible force on the other charges. So, 'q' is too vanishingly small.

Consider a point charge which is positive i.e., $+Q$ as shown in figure. The test charge or the static charge ($+q$) is located at 'S' at a distance 'r' from point charge $+Q$.

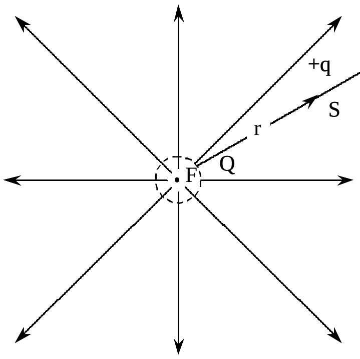


Figure: Electric Field at a Point Charge +Q

According to Coulomb's law, the test charge experiences a force and an electric field is produced around a charge $+Q$, where the force acts and if any charge is brought in this region, it also experiences a force. The force is repulsive as field and test charge both are positive and this force is directed radially outwards.

The force experienced by the test charge is given as,

$$\vec{F} = \vec{a}_r\left[\frac{Qq}{4\pi\epsilon_o r^2}\right]$$

Where,

$\vec{a}_r$ = A unit vector directed away from point charge.

As Electric field,

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\Rightarrow \quad \boxed{\vec{E} = \frac{Q}{4\pi\epsilon_o r^2}\vec{a}_r \text{ N/C}}$$

Q2. Define and explain the following terms,

(i) Electrostatic fields

(ii) Electric field intensity

(iii) Electric potential

(iv) Potential gradient.

Answer :

(i) Electrostatic Fields

The electric field produced by a static charge distribution is said to be electrostatic field. Basically, this field is conservative in nature i.e., it obeys conservative property. It means that, the work done in carrying a unit charge around any closed path within the field is zero. These fields are governed by two basic laws namely, Coulomb's law and Gauss's law.

(ii) Electric Field Intensity (E)

Electric Field Intensity or strength (EFI) is defined as the force exerted per unit charge at a point in the vicinity of field.

i.e., $\bar{E} = \dfrac{\bar{F}}{Q'}$

Where,

$\bar{F}$ = Force exerted between the charges.

$Q^{'}$ = Charge present at the point of consideration.

$$\Rightarrow \quad \bar{E} = \frac{\frac{Q\,Q^{'}}{4\pi\in_0 R^2}\bar{a}_R}{Q^{'}} = \frac{Q}{4\pi\in_0 R^2}\bar{a}_R$$

Where,

R = Distance between the charges Q and $Q^{'}$

As $E^{'}$ is inversely proportional to the square of the distance, the field due to a point is non-uniform in nature.

EFI due to a point charge distribution is given by,

$$E^{'} = \frac{1}{4\pi\in_0}\sum_{K=1}^{n}\frac{Q_K}{R_K^2}\bar{a}_K$$

(iii) Electric Potential (V)

Electric potential is defined as the work done per unit charge in bringing a point charge from infinity or a zero reference point into the vicinity of an electric field.

i.e., $$W = -Q\int_{\infty}^{r}\vec{E}.d\vec{l} \quad \ldots (1)$$

We know that, $$V = \frac{W}{Q} \quad \ldots (2)$$

Substituting equation (2) in equation (1), we get,

$$V = \frac{-Q\int_{\infty}^{r}\vec{E}.d\vec{l}}{Q}$$

$$\Rightarrow \quad V = -\int_{\infty}^{r}\vec{E}.d\vec{l}$$

$$\Rightarrow \quad V = -\int_{\infty}^{r}\frac{Q}{4\pi\in_0 R^2}\bar{a}_R.\bar{a}_R dr$$

$$\left[\because E = \frac{Q}{4\pi\in_0 R^2}\bar{a}_r \text{ and } d\bar{l} = \bar{a}_R dr\right]$$

Integrating above equation, we get,

$$\Rightarrow \quad V = \frac{-Q}{4\pi\in_0}\left[\frac{R^{-2+1}}{(-2+1)}\right]_{\infty}^{r}$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\in_0}\left[\frac{1}{R}\right]_{\infty}^{r}$$

$$\therefore \quad V = \frac{Q}{4\pi\in_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{Q}{4\pi\in_0 r}$$

Hence, it is a scalar quantity with units of J/C or V.

(iv) Potential Gradient

In general sense, the term gradient is a measure of rise in elevation between two point on a slope of the hill. Similarly, in electric fields, the gradient of potential or potential gradient is a measure of potential rise between two points.

So, the potential gradient is defined as the gradient of potential function and it is given by negative of electric field intensity.

i.e., $$\nabla V = -\bar{E}$$

or $$\bar{E} = -\nabla V$$

On applying curl on both sides of above equation, the Maxwell's curl equation is obtained as,

$$\nabla \times \bar{E} = \nabla \times (-\nabla V)$$

$$\nabla \times \bar{E} = -\nabla \times \nabla V$$

$$\therefore \quad \nabla \times \bar{E} = 0$$

1.2 Coulomb's Law

Q3 State and explain Coulomb's law for the vector force between two point charger in free space.

Feb.-08, Set-2, Q1(a) M[6]
Nov.-07, Set-2, Q1(a) M[6]
Nov.-07, Set-4, Q1(a) M[6]
Feb.-07, Set-2, Q1(a) M[4]
Nov.-06, Set-2, Q1(a) M[4]
Nov.-06, Set-3, Q1(a) M[4]

OR

State and explain Coulomb's Law.

May-05, Set-3, Q1(a)

Answer :

Coulomb's law states that, "the force between two charged bodies is directly proportional to the product of their charges and inversely proportional to the square of distance between them". Provided that, the sizes of the charged bodies are quite negligible when compared with the distance between them, due to which those charged bodies are usually termed as point charges.

Consider two point charges Q_a and Q_b separated by a distance R. Then from Coulomb's law, the force between Q_a and Q_b can be determined as,

$$F \propto Q_a Q_b \text{ and}$$

$$F \propto \frac{1}{R^2}$$

$$\Rightarrow \quad F = K\frac{Q_a Q_b}{R^2}$$

Where,

K = Constant of proportionality

$$= \frac{1}{4\pi\epsilon_0} \quad \text{[For free space]}$$

$$\therefore \quad F = \frac{1}{4\pi\epsilon_0}\times\frac{Q_a Q_b}{R^2} = \frac{Q_a Q_b}{4\pi\epsilon_0 R^2}\ \text{N}$$

The above expression gives the magnitude of force exerted on Q_a by Q_b or on Q_b by Q_a. If the magnitude of force is a negative value then, it indicates that the force acting between the charges is an attracting force and if it is positive value then, it indicates that the force acting between the charges is a repulsive force. This is because of the fact that the like charges repels and unlike one attracts, each other. Either kind of force will act along the straight line joining the charges.

Hence, Coulomb's law can be defined in its vector form as,

$$\bar{F} = \frac{Q_a Q_b}{4\pi\epsilon_0 R^2}\bar{a}$$

Where, $\bar{a}$ is the unit vector which specifies the direction of force between the charges.

So, the force exerted on Q_a by Q_b can be expressed as,

$$\bar{F}_{ba} = \frac{Q_a Q_b}{4\pi\epsilon_0 R^2}\bar{a}_{ba}$$

Where,

$$\bar{a}_{ba} = \frac{\bar{R}_{ba}}{|\bar{R}_{ba}|} = \frac{\bar{R}_{ba}}{R}$$

and $\quad \bar{R}_{ba} = \bar{R}_a - \bar{R}_b$

The vectors $\bar{R}_a$ and $\bar{R}_b$ indicates the location of the charges Q_a and Q_b respectively.

Coulomb's law obeys linearity and also the superposition principle due to which, the force exerted on a point charge by 'n' number of different point charges can be given by the vector summation of all 'n' individual forces acting on that point charge i.e.,

$$\bar{F}_{\text{total}} = \sum_{i=1}^{n}\bar{F}_i$$

Q4. Two small identical conducting spheres have charge of 2 nC and – 0.5 nC respectively. When they are placed 4 cm apart, what is the force between them? If they are brought into contact and then separated by 4 cms, what is the force between them?

May-05, Set-3, Q1(b)

Answer :

Given data,

Charges of conducting spheres,

Q_a = 2 nC
= 2 × 10^{-9} C

Q_b = – 0.5 nC
= – 0.5 × 10^{-9} C

Distance of separation, R = 4 cm
= 4 × 10^{-2} m

Force between charges, F = ?

According to Coulomb's law, force between two charges Q_a and Q_b displaced by 'R'm is,

$$F = \frac{Q_a Q_b}{4\pi\epsilon_0 R^2}$$

$$= \frac{2\times10^{-9}\times(-0.5\times10^{-9})}{4\pi\times8.854\times10^{-12}\times(4\times10^{-2})^2}$$

= – 5.62 × 10^{-6} N
= – 5.62 μN

The negative value of force indicates that it is an attractive force which is quite obvious for opposite polarity charges.

Now, when both the charges are brought into contact, the charge of both the spheres equals to their average value of charge before.

i.e., $$Q_a' = Q_b' = \frac{Q_a + Q_b}{2}$$

$$= \frac{2\times10^{-9} - 0.5\times10^{-9}}{2}$$

= 0.75 × 10^{-9} C
= 0.75 nC

∴ Force between the charges after separated by 4 cm,

$$F' = \frac{Q_1' Q_2'}{4\pi\epsilon_0 R^1}$$

$$= \frac{0.75\times10^{-9}\times0.75\times10^{-9}}{4\pi\times8.854\times10^{-12}\times(4\times10^{-2})^2}$$

= 3.16 × 10^{-6} N
= 3.16 μN

The positive value of force indicates that, it is a repulsive force which is quite obvious for same polarity charges.

Hence, by this it can be said that, "if an attracting force exists between two charges and if they are brought into contact and then separated, then the force between the chargers changes into a repulsive force due to neutralization of charges".

Q5. Explain the superposition principle governing the forces between charges at rest.

Feb.-07, Set-3, Q1(a) M[4]

Answer :

Consider two point charges Q_1 and Q_2 be located at points P_1 and P_2 respectively.

According to Coulomb's law, the force exerted on charge Q_1 due to charge Q_2 is,

$$\overline{F}_{21} = \frac{Q_1 Q_2}{4\pi\in_0 |\overline{R}_1 - \overline{R}_2|^2}\overline{a}_{21}$$

Where, the vectors $\overline{R}_1$ and $\overline{R}_2$ indicates the location of the charges Q_1 and Q_2 respectively, $\overline{a}_{21}$ is the unit vector which indicates the direction force (i.e., from Q_2 to Q_1).

Let us consider another charge Q_3 at point P_3.

Now, the force exerted on charge Q_1 due to charge Q_3 is,

$$\overline{F}_{31} = \frac{Q_1 Q_3}{4\pi\in_0 |\overline{R}_1 - \overline{R}_3|^2}\overline{a}_{31}$$

According to superposition principle governing i.e., the forces between charges at rest.

We have, the total force exerted on charge Q_1 due to the charges Q_2 and Q_3 is the vectorial sum of the force exerted on Q_1 due to individual point charge in the absence of other charges i.e.,

$$\overline{F_1} = \overline{F}_{21} + \overline{F}_{31}$$

$$= \frac{Q_1 Q_2}{4\pi\in_0 |\overline{R}_1 - \overline{R}_2|^2}\overline{a}_{21} + \frac{Q_1 Q_3}{4\pi\in_0 |\overline{R}_1 - \overline{R}_3|^2}\overline{a}_{31}$$

$$= \frac{Q_1}{4\pi\in_0}\left[\frac{Q_2}{|\overline{R}_1 - \overline{R}_2|^2}\overline{a}_{21} + \frac{Q_3}{|\overline{R}_1 - \overline{R}_3|^2}\overline{a}_{31}\right]$$

In general according to superposition principle, the force exerted on charge Q_1 due to 'n – 1' point charges Q_2, Q_3, ..., Q_n is,

$$\overline{F_1} = \frac{Q_1}{4\pi\in_0}\left[\frac{Q_2}{|\overline{R}_1 - \overline{R}_2|^2}\overline{a}_{21} + \frac{Q_3}{|\overline{R}_1 - \overline{R}_3|^2}\overline{a}_{31} + ...+ \frac{Q_{n-1}}{|\overline{R}_1 - \overline{R}_{n-1}|^2}\overline{a}_{(n-1)1} + \frac{Q_n}{|\overline{R}_1 - \overline{R}_n|^2}\overline{a}_{n1}\right]$$

$$= \frac{Q_1}{4\pi\in_0}\sum_{k=2}^{n}\frac{Q_k}{|\overline{R}_1 - \overline{R}_k|^2}\overline{a}_{k1}$$

Above formula can be generalized to calculate the force exerted on charge Q_i due to all the other charges as,

$$\overline{F_i} = \frac{Q_i}{4\pi\in_0}\sum_{\substack{k=1\\k\neq i}}^{n}\frac{Q_k}{|\overline{R}_i - \overline{R}_k|^2}\overline{a}_{ki}$$

Q6. Show that the force on a point charge anywhere with in a circular ring of uniform charge density is zero provided the point charge remains in the plane of the ring.

Nov.-04, Set-2, Q1(a)

Answer :

Consider a ring in the *y-z* plane as shown in figure (1).

Let, λ – Line charge density in C/m.

F – Force on a point charge in Newtons.

A – Point in the plane of the ring

dQ – Differential charge element on the ring.

D – Distance between the point A and the differential charge element dQ

r – Radius of the ring.

y – Distance of the point on the axis of the ring.

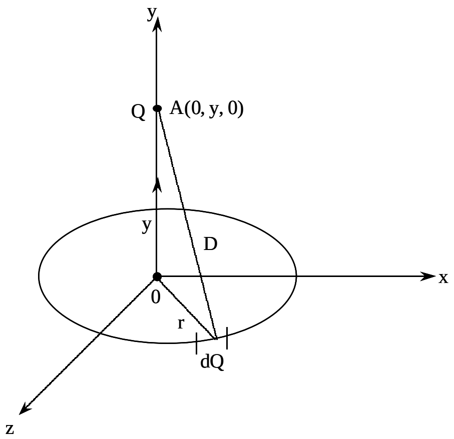


Figure (1)

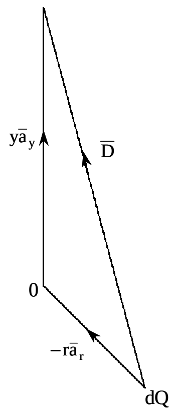


Figure (2)

The force acting on Q due to the differential charge dQ is given by,

$$F = \int \frac{QdQ}{4\pi\epsilon_0 D^2} \bar{a}_D \quad \text{... (1)}$$

From figure (2),

$$D^2 = r^2 + y^2 \quad \text{... (2)}$$

and

$$\bar{a}_D = \frac{-r\bar{a}_r + y\bar{a}_y}{\sqrt{r^2 + y^2}} \quad \text{... (3)}$$

Substituting equations (2) and (3), in equation (1), we get,

$$F = \int \frac{QdQ}{4\pi\epsilon_0 (r^2 + y^2)} \left[\frac{-r\bar{a}_r + y\bar{a}_y}{\sqrt{r^2 + y^2}} \right]$$

$$F = \int \frac{QdQ\,(-r\bar{a}_r + y\bar{a}_y)}{4\pi\epsilon_0 (r^2 + y^2)^{3/2}} \quad \text{... (4)}$$

The radial component is absent due to the symmetry of the figure. Therefore equation (4) can now be written as,

$$F = \int_{Q=0}^{2\pi} \frac{Q\lambda r dQ\, y\bar{a}_y}{4\pi\epsilon_0 (r^2+y^2)^{3/2}} \quad \text{... (5)}$$

Integrating equation (5) w.r.t dQ, we get,

$$F = \frac{Q\lambda r y\bar{a}_y}{4\pi\epsilon_0 (r^2+y^2)^{3/2}}[Q]_0^{2\pi}$$

$$F = \frac{Q\lambda r y\bar{a}_y}{4\pi\epsilon_0 (r^2+y^2)^{3/2}}(2\pi-0)$$

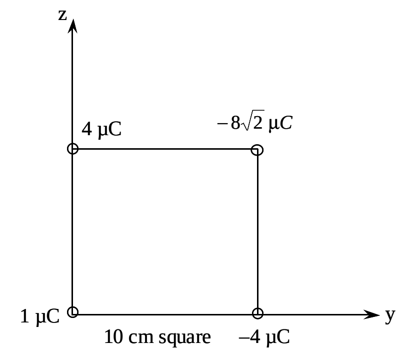
$$F = \frac{Q\lambda r y\bar{a}_y}{2\epsilon_0 (r^2+y^2)^{3/2}}$$

Since, it is given that A is a point on the ring then the distance of the point from origin will be zero i.e., $y = 0$.

$$F = \frac{Q\lambda r \times 0 \times \bar{a}_y}{2\epsilon_0 (r^2+y^2)^{3/2}}$$

$$\therefore \quad F = 0$$

Q7. Charges are located in free space as shown in figure. Find the force experienced by the 1 μC charge.

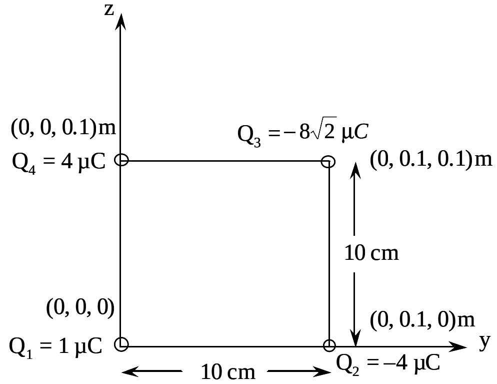


Figure

Feb.-07, Set-3, Q1(c) M[6]

Answer :

Given data,



Figure

Let the charges 1 μC, – 4 μC, $-8\sqrt{2}\,\mu C$ and 4 μC be denoted by Q_1, Q_2, Q_3 and Q_4 respectively.

According to the superposition principle governing the forces between charges at rest we have,

The force experienced by, Q_1 = 1 μC

$$\overline{F_1} = \frac{Q_1}{4\pi\epsilon_0}\sum_{k=2}^{4}\frac{Q_k}{|\overline{R}_1-\overline{R}_k|^2}\overline{a}_{k1}$$

$$= \frac{Q_1}{4\pi\epsilon_0}\left[\frac{Q_2}{|\overline{R}_1-\overline{R}_2|^2}\overline{a}_{21}+\frac{Q_3}{|\overline{R}_1-\overline{R}_3|^2}\overline{a}_{31}+\frac{Q_4}{|\overline{R}_1-\overline{R}_4|^2}\overline{a}_{41}\right]$$

$$\overline{R}_1-\overline{R}_2 = (0-0)\overline{a}_x+(0-0.1)\overline{a}_y+(0-0)\overline{a}_z$$

$$= -0.1\,\overline{a}_y$$

$$|\overline{R}_1-\overline{R}_2| = \sqrt{(-0.1)^2} = 0.1 \text{ m}$$

$$\therefore \quad \overline{a}_{21} = \frac{\overline{R}_1-\overline{R}_2}{|\overline{R}_1-\overline{R}_2|}$$

$$= \frac{-0.1\overline{a}_y}{0.1}$$

$$= -\overline{a}_y$$

$$\overline{R}_1-\overline{R}_3 = (0-0)\overline{a}_x+(0-0.1)\overline{a}_y+(0-0.1)\overline{a}_z$$

$$= -0.1\,\overline{a}_y - 0.1\,\overline{a}_z$$

$$|\overline{R}_1-\overline{R}_3| = \sqrt{(-0.1)^2+(-0.1)^2}$$

$$= 0.1\sqrt{2} \text{ m}$$

$$\overline{a}_{31} = \frac{\overline{R}_1-\overline{R}_3}{|\overline{R}_1-\overline{R}_3|}$$

$$= \frac{-0.1\overline{a}_y-0.1\overline{a}_z}{0.1\times\sqrt{2}}$$

$$= \frac{-1}{\sqrt{2}}(\overline{a}_y+\overline{a}_z)$$

$$\overline{R}_1-\overline{R}_4 = (0-0)\,\overline{a}_x+(0-0)\,\overline{a}_y+(0-0.1)\,\overline{a}_z$$

$$= -0.1\,\overline{a}_z$$

$$|\overline{R}_1-\overline{R}_4| = \sqrt{(-0.1)^2}$$

$$= 0.1 \text{ m}$$

$$\overline{a}_{41} = \frac{\overline{R}_1-\overline{R}_4}{|\overline{R}_1-\overline{R}_4|} = \frac{-0.1\overline{a}_z}{0.1}$$

$$= -\overline{a}_z$$

$$\therefore \quad \bar{F}_1 = \frac{1\times10^{-6}}{4\pi\times8.854\times10^{-12}}\left[\frac{-4\times10^{-6}}{(0.1)^2}\times(-\bar{a}_y)+\frac{-8\sqrt{2}\times10^{-6}}{(0.1\sqrt{2})^2}\times\frac{(-1)}{\sqrt{2}}(\bar{a}_y+\bar{a}_z)+\frac{4\times10^{-6}}{(0.1)^2}\times(-\bar{a}_z)\right]$$

$$= \frac{1\times10^{-6}\times4\times10^{-6}}{4\pi\times8.854\times10^{-12}}\left[100\,\bar{a}_y+100\bar{a}_y+100\bar{a}_z-100\,\bar{a}_z\right]$$

$$= \frac{1}{8.854\pi}\,200\,\bar{a}_y$$

$$= 7.1902\,\bar{a}_y \text{ N.}$$

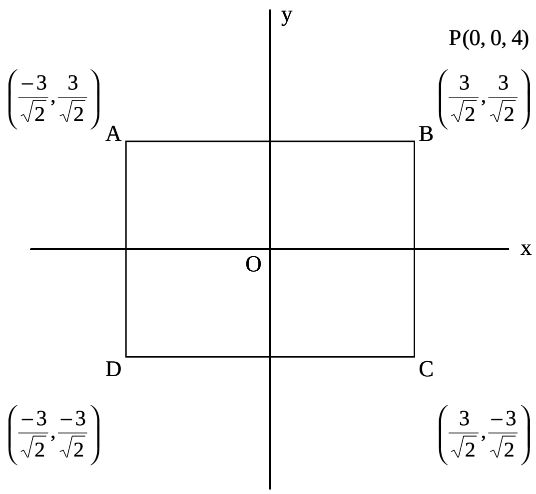
Q8. 4-point charges of 500 μC each are placed at the corners of a square of $3\sqrt{2}$ m side. The square is located in the z = 0 plane between x = $\pm\frac{3}{\sqrt{2}}$ m and y = $\pm\frac{3}{\sqrt{2}}$ m in free space. Find the force on a point charge of 30 μC at (0, 0, 4) m.

Feb.-08, Set-3, Q1(c) M[4]

Answer :

Given that 4-point charges are placed in a square of side $3\sqrt{2}$ m between x and y.

∴ It can be shown as,



Figure

To find force 'F' on a point charge 'P' of 30 μC at (0, 0, 4) m.

$$AP = \sqrt{(3/\sqrt{2})^2+(3/\sqrt{2})^2+(4)^2}$$

$$= \sqrt{(9/2)+(9/2)+(16)}$$

$$= \sqrt{\frac{18+32}{2}}$$

$$= \sqrt{25}$$

$$BP = \sqrt{(3/\sqrt{2})^2+(3/\sqrt{2})^2+(4)^2} = \sqrt{25} = DP = CP$$

Force due to point charge at 'A' is,

$$F_{AP} = \frac{1}{4\pi\in_o}.\frac{30\times10^{-6}}{25}\, iAP$$

$$= \frac{1}{4\pi\in_o}.\frac{30\times10^{-6}}{25}\, i\left[\frac{(3/\sqrt{2})x-(3/\sqrt{2})y+4z}{\sqrt{(3/\sqrt{2})^2+(3/\sqrt{2})^2+(4)^2}}\right]$$

$$= 9\times10^9\times\frac{30}{25}\times10^{-6}\left[\frac{(3/\sqrt{2})\,ix-(3/\sqrt{2})\,iy+4iz}{\sqrt{25}}\right]$$

Similarly,

$$iBP = \frac{(-3/\sqrt{2})\,ix-(3/\sqrt{2})\,iy+4iz}{\sqrt{25}}$$

$$iCP = \frac{(-3/\sqrt{2})\,ix+(3/\sqrt{2})\,iy+4iz}{\sqrt{25}}$$

$$iDP = \frac{(3/\sqrt{2})\,ix+(3/\sqrt{2})\,iy+4iz}{\sqrt{25}}$$

$$\therefore \quad F = F_{AP}+F_{BP}+F_{CP}+F_{DP}$$

$$F = 9\times10^9\times\frac{30\times10^{-6}}{25}\,[iAP+iBP+iCP+iDP]$$

$$= 9\times10^9\times\frac{30\times10^{-6}}{25\sqrt{25}}\left[\begin{matrix}(3/\sqrt{2})\,ix-(3/\sqrt{2})\,iy+4\,iz-(3/\sqrt{2})\,ix-(3/\sqrt{2})\,iy+4iz-\\(3/\sqrt{2})\,ix+(3/\sqrt{2})\,iy+4iz+(3/\sqrt{2})\,ix+(3/\sqrt{2})\,iy+4iz\end{matrix}\right]$$

$$= \frac{9\times10^9\times30\times10^{-6}}{(25)^{3/2}}\times16\,iz$$

$$= \frac{9\times10^3\times30}{(25)^{3/2}}\times16\,iz$$

$$\therefore \quad \boxed{F = \frac{27\times10^4}{125}\times16\,iz}$$

1.3 Electric Field Intensity Due to a Line Charge

Q9. Find the E at any point due to a line charge of density λ C/m and length L meter.

Nov.-04, Set-1, Q1(a)

Answer :

Consider a uniformly charged wire of length L meter. Let λ be the linear charge density in Coulombs/meter. Let A be any point at a distance 'd' from the wire.

Consider a differential element dy at a distance x m from one end of the wire as shown in figure (1).

The electric field intensity due to differential charge element $\lambda\,dx$ is given by,

$$dE = \frac{\lambda\,dx}{4\pi\in_0 y^2} \quad \ldots (1)$$

The elemental field dE can be resolved into two components i.e., the horizontal component along X-axis and the vertical component along Y-axis is given as,

$$\therefore \quad dE_h = dE \cos\phi \quad \ldots (2)$$

$$dE_v = dE \sin\phi \quad \ldots (3)$$

Substituting equation (1) in equation (2), we get,

$$dE_h = \frac{\lambda\, dx}{4\pi\epsilon_0 y^2}\cos\phi$$

$$= \frac{\lambda \cos\phi}{4\pi\epsilon_0 y^2}dx \quad \ldots (4)$$

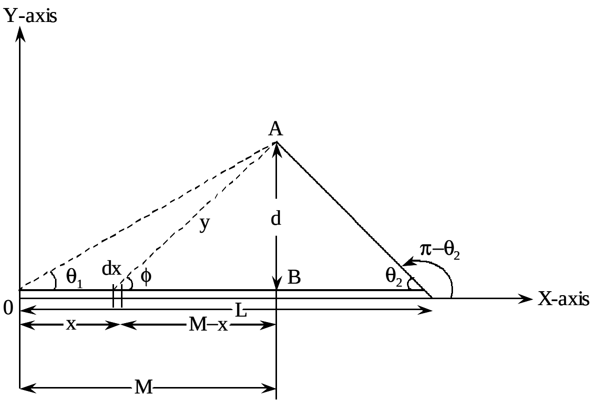


Figure (1)

Consider Δ^{le} ABC as shown in figure (2),

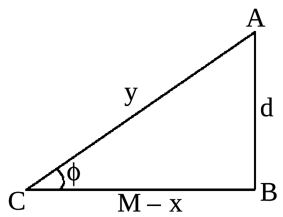


Figure (2)

From figure (2),

$$\cot\phi = \frac{M - x}{d}$$

$$M - x = d \cot\phi \quad \ldots (5)$$

Differentiating equation (5), we get,

$$0 - dx = -d \operatorname{cosec}^2\phi \, d\phi$$

$$dx = d \operatorname{cosec}^2\phi \, d\phi \quad \ldots (6)$$

From figure (2),

$$\operatorname{cosec}\phi = \frac{y}{d}$$

$$y = d \operatorname{cosec}\phi \quad \ldots (7)$$

Substituting equations (6) and (7) in equation (4), we get,

$$dE_h = \frac{\lambda \cos\phi \, d \operatorname{cosec}^2\phi \, d\phi}{4\pi\epsilon_0 d^2 \operatorname{cosec}^2\phi}$$

$$dE_h = \frac{\lambda \cos\phi \, d\phi}{4\pi\epsilon_0 d} \quad \ldots (8)$$

In order to get the electric field over the whole length of the wire, integrating equation (8),

$$E_h = \int_{\phi=\theta_1}^{\pi-\theta_2} \frac{\lambda \cos\phi \, d\phi}{4\pi\epsilon_0 d}$$

$$E_h = \frac{\lambda}{4\pi\epsilon_0 d}\int_{\theta_1}^{\pi-\theta_2} \cos\phi \, d\phi$$

$$E_h = \frac{\lambda}{4\pi\epsilon_0 d}\left[\sin\phi\right]_{\theta_1}^{\pi-\theta_2}$$

$$E_h = \frac{\lambda}{4\pi\epsilon_0 d}\left[\sin\theta_2 - \sin\theta_1\right]$$

Considering equations (3) i.e.,

$$dE_v = dE \sin\phi$$

$$dE_v = \frac{\lambda\, dx \sin\phi}{4\pi\epsilon_0 y^2}$$

$$dE_v = \frac{\lambda \sin\phi \, dx}{4\pi\epsilon_0 y^2} \quad \ldots (9)$$

Substituting equations (6) and (7) in equation (9), we get,

$$dE_v = \frac{\lambda \sin\phi \, d \operatorname{cosec}^2\phi \, d\phi}{4\pi\epsilon_0 d^2 \operatorname{cosec}^2\phi}$$

$$dE_v = \frac{\lambda \sin\phi \, d\phi}{4\pi\epsilon_0 d}$$

In order to get the vertical component of E along the length of the wire, integrating equation (10),

$$dE_v = \frac{\lambda}{4\pi\epsilon_0 d}\int_{\theta_1}^{\pi-\theta_2} \sin\phi \, d\phi$$

$$= \frac{\lambda}{4\pi\epsilon_0 d}\left[\cos\theta_1 + \cos\theta_2\right]$$

Q10. A straight line of length *l* m in free space has a uniform charge density of λ C/m. P is a point on the perpendicular bisector of the line charge, at a distance y m from the line. Find the electric field at P.

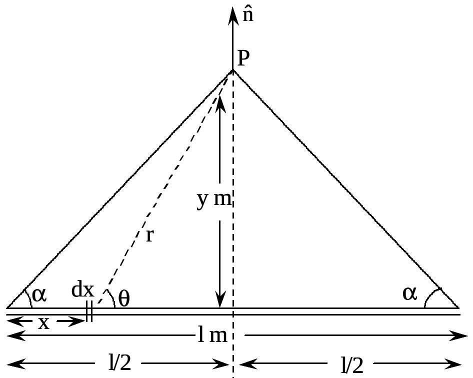
Feb.-08, Set-2, Q1(b) M[10]

Nov.-07, Set-4, Q1(b) M[10]

Answer:

Consider a uniformly charged wire of length *l* m as shown in figure. The point *P* is on the perpendicular bisector of the line charge, at a distance *y* m from the line.

Let $\hat{n}$ be the unit vector at the point *P*, perpendicular to the line. Let *dx* be the elemental length at a distance *x* from the wire. Let *dE* be the electric field due to the charge λdx [λ is the linear charge density].



Figure

We know that any vector can be resolved into two components i.e., the horizontal component and the vertical component. Similarly the vector *dE* can be resolved in to the horizontal component dE_x and the vertical component dE_y. Since the point *P* is along the perpendicular bisector of the line, the horizontal component dE_x will be absent or zero.

$$\therefore \quad dE = dE_y$$

$$dE_y = dE \sin\theta$$

$$dE_y = \frac{\lambda\, dx}{4\pi\epsilon_o r^2}\sin\theta \qquad \text{... (1)}$$

From figure,

$$\frac{l-x}{2} = y\cot\theta \qquad \text{... (2)}$$

Differentiating equation (2), we get,

$$-dx = -y\,\text{cosec}^2\theta\, d\theta.$$

$$dx = y\,\text{cosec}^2\theta\, d\theta. \qquad \text{... (3)}$$

Also, $\quad r = y\,\text{cosec}\theta \qquad \text{... (4)}$

Substituting equation (3) and equation (4) in equation (1) we get,

$$dE_y = \frac{\lambda\, y\,\text{cosec}^2\theta\, d\theta}{4\pi\epsilon_o (y\,\text{cosec})^2}\sin\theta$$

$$= \frac{\lambda\, y\,\text{cosec}^2\theta\, d\theta}{4\pi\epsilon_o y^2\,\text{cosec}^2\theta}\sin\theta$$

$$dE_y = \frac{\lambda\sin\theta\, d\theta}{4\pi\epsilon_o y}$$

$$\therefore \quad E = E_y = \int_{\alpha}^{\pi-\alpha}\frac{\lambda\sin\theta\, d\theta}{4\pi\epsilon_o y}$$

$$= \frac{\lambda}{4\pi\epsilon_o y}\, 2\cos\alpha$$

$$\therefore \quad E = \frac{\lambda\cos\alpha}{2\pi\epsilon_o y}\hat{n}$$

Q11. Find the electric field at P if the line charge length *l* extends to infinity.

Nov.-07, Set-4, Q1(c) M[2]

Answer :

The electric field at point '*P*' is given by,

$$E = \frac{\lambda\cos\alpha}{2\pi\epsilon_0 y}\hat{n}$$

When the length '*l*' extends to infinity i.e., for infinitely long conductors,

$\alpha = 0$ and hence $\cos\alpha \Rightarrow \cos 0° = 1$

$$\boxed{\therefore\ E = \hat{n}\frac{\lambda}{2\pi\epsilon_0 y}}$$

Where,

$\hat{n}$ = Unit normal vector at point '*P*'.

Q12. A uniform line charge λ_L = 25 nC/m lies on the line x = –3 and z = 4 m in free space. Find the electric field intensity at a point (2, 5, 3) m.

Nov.-04, Set-4, Q1(b)

Answer :

Derivation

For anwser refer Unit-I, Q40.

In general $\overline{E}$ can be written as,

$$\overline{E} = \frac{\rho_L}{2\pi\epsilon_0 R}\overline{a}_r$$

Where,

R – Distance between the line and point charges.

$\bar{a}_R$ – Unit vector directed from line charge to point charge.

Problem

Given data,

Uniform line charge, $\rho_L = \lambda_L = 25$ nC/m

$= 25 \times 10^{-9}$ C/m

ρ_L lies on the line $x = -3$ m and $z = 4$ m in free space

At point $P = (2, 5, 3)$ m

Electric field, $\bar{E} = ?$

The line charge lies in x-z plane and extends towards infinity without intersecting y-axis, it is parallel to y-axis and hence y-component does not exists in the field.

$$\therefore \quad \bar{R} = (2-(-3))\,\bar{a}_x + (3-4)\,\bar{a}_z$$

$$= 5\,\bar{a}_x - \bar{a}_z$$

$$\therefore \quad R = |\bar{R}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$$

$$\therefore \quad \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{5\bar{a}_x - \bar{a}_z}{\sqrt{26}}$$

$\therefore$ Electric field intensity at point P,

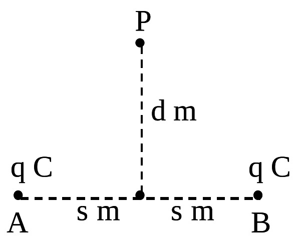
$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 R}\,\bar{a}_R$$

$$= \frac{25\times10^{-9}}{2\pi\times8.854\times10^{-12}\times\sqrt{26}} \times \frac{5\bar{a}_x - \bar{a}_z}{\sqrt{26}}$$

$$= 17.284\,(5\,\bar{a}_x - \bar{a}_z)$$

$$\therefore \quad \bar{E} = 86.42\,\bar{a}_x - 17.284\,\bar{a}_z$$

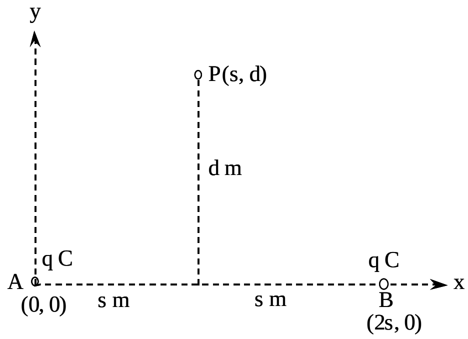
Q13. Figure shows two charges at points A and B in free space. Find the electric field at point P due to these charges. Is the result consistent with what may be expected if d>>s?



Figure

Nov.-07, Set-2, Q1(b) M[6]
Feb.-07, Set-2, Q1(b) M[6]
Nov.-06, Set-2, Q1(b) M[6]
Nov.-06, Set-3, Q1(b) M[6]

Answer :



Electric field at point '*P*' due to charges at points *A* and *B*.

$$\bar{E} = \frac{q}{4\pi\epsilon_0 |\bar{R}_P - \bar{R}_A|^2}\bar{a}_{AP} + \frac{q}{4\pi\epsilon_0 |\bar{R}_P - \bar{R}_B|^2}\bar{a}_{BP}$$

Let point '*A*' be located at origin.

$$\Rightarrow \quad \bar{R}_P - \bar{R}_A = (s-0)\,\bar{a}_x + (d-0)\,\bar{a}_y$$

$$= s\,\bar{a}_x + d\,\bar{a}_y$$

$$\therefore \quad |\bar{R}_P - \bar{R}_A| = \sqrt{s^2 + d^2}$$

$$\therefore \quad \bar{a}_{AP} = \frac{\bar{R}_P - \bar{R}_A}{|\bar{R}_P - \bar{R}_A|} = \frac{s\bar{a}_x + d\,\bar{a}_y}{\sqrt{s^2+d^2}}$$

Similarly,

$$\bar{R}_P - \bar{R}_B = (s-2s)\,\bar{a}_x + (d-0)\,\bar{a}_y$$

$$= -s\,\bar{a}_x + d\,\bar{a}_y$$

$$\therefore \quad |\bar{R}_P - \bar{R}_B| = \sqrt{s^2 + d^2}$$

$$\bar{a}_{BP} = \frac{\bar{R}_P - \bar{R}_B}{|\bar{R}_P - \bar{R}_B|}$$

$$= \frac{-s\bar{a}_x + d\,\bar{a}_y}{\sqrt{s^2+d^2}}$$

$$\therefore \quad \bar{E} = \frac{q}{4\pi\epsilon_0\left(\sqrt{s^2+d^2)}\right)^2} \times \left(\frac{s\bar{a}_x + d\,\bar{a}_y}{\sqrt{s^2+d^2}}\right) + \frac{q\times(-s\bar{a}_x + d\,\bar{a}_y)}{4\pi\epsilon_0\left(\sqrt{s^2+d^2}\right)^2\left(\sqrt{s^2+d^2)}\right)}$$

$$= \frac{q}{4\pi\epsilon_0\,(s^2+d^2)^{3/2}}\left(s\bar{a}_x + d\,\bar{a}_y - s\bar{a}_x + d\,\bar{a}_y\right)$$

$$= \frac{q}{4\pi\epsilon_0\,(s^2+d^2)^{3/2}} \times 2d\,\bar{a}_y$$

$$= \frac{q}{2\pi\epsilon_0 d^3\left(1+\left(\frac{s}{d}\right)^2\right)^{3/2}} d\,\bar{a}_y$$

$$\therefore \bar{E} = \frac{q}{2\pi\epsilon_0 d^2\left(1+\left(\frac{s}{d}\right)^2\right)^{3/2}} \bar{a}_y$$

If $d >> s$, then ratio s/d can be neglected.

$$\Rightarrow \quad \bar{E} = \frac{q \quad \bar{a}_y}{2\pi\epsilon_0 d^2 \, (1+0)^{3/2}}$$

$$\therefore \bar{E} = \frac{q}{2\pi\epsilon_0 d^2}\bar{a}_y$$

Yes, the result is consistent with what may be expected if $d >> s$.

Q14. Two point charge –q and q/2 are situated at the origin and at the point (a, 0, 0) respectively. At what point does the electric field vanish?

Nov.-04, Set-3, Q1(a)

Answer :

Given data,

Point charge at (0, 0, 0), $Q_1 = -q$

Point charge at (a, 0, 0), $Q_2 = q/2$

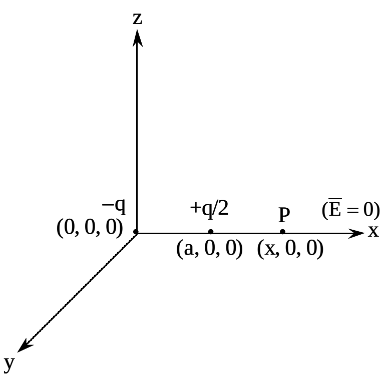
point, P = ?

Where, $E = 0$

As both the point charges lie on X-axis only, the point where electric field vanishes will also lie on X-axis. So, let the point P be (x, 0, 0)

$\therefore$ The magnitudes of electric field intensities at 'P' due to Q_1 and Q_2 will be equal.

i.e., $|\bar{E}_1| = |\bar{E}_2|$



$$\Rightarrow \left|\frac{Q_1}{4\pi\epsilon_0 R_1^2}\right| = \left|\frac{Q_2}{4\pi\epsilon_0 R_2^2}\right|$$

$$\Rightarrow \left|\frac{Q_1}{R_1^2}\right| = \left|\frac{Q_2}{R_2^2}\right|$$

$$\Rightarrow \frac{q}{x^2} = \frac{q/2}{(x-a)^2}$$

$$\Rightarrow \frac{1}{x^2} = \frac{1}{2(x^2-2ax+a^2)}$$

$\Rightarrow 2x^2 - 4ax + 2a^2 = x^2$

$\Rightarrow x^2 - 4ax + 2a^2 = 0$

$$\therefore \quad x = \frac{-(-4a)\pm\sqrt{(-4a)^2-(4\times1\times2a^2)}}{2\times1}$$

$$= \frac{4a\pm\sqrt{16a^2-8a^2}}{2}$$

$$= \frac{4a\pm\sqrt{8a^2}}{2}$$

$$= \frac{2a(2\pm\sqrt{2})}{2} = a(2\pm\sqrt{2})$$

$= 0.586\,a$ (or) $3.414\,a$.

The point $x = a(2-\sqrt{2}) = 0.568\,a$ lies in the region between the charges nearer to the midpoint.

As $|Q_1| = 2|Q_2|$, the point cannot lie near the midpoint, hence the point 'P' must lie away from the midpoint of the charges.

$\therefore \quad x = 3.414\,a$ is the required point.

Hence, P = (3.414 a, 0, 0) is the required point where the electric field vanishes.

Q15. Potential distributions are given by $V=4/(x^2+y+z^2)$. Find the expression for E.

Nov.-05, Set-1, Q2(b) M[8]

Answer :

Given potential distributions,

$$V = \frac{4}{x^2+y+z^2}$$

Magnitude of electric field intensity, E = ?

We know that,

$$\overline{E} = -\nabla V$$

$$= -\left(\overline{a}_x \frac{\partial}{\partial x} + \overline{a}_y \frac{\partial}{\partial y} + \overline{a}_z \frac{\partial}{\partial z}\right)V$$

$$= -\left(\overline{a}_x \frac{\partial V}{\partial x} + \overline{a}_y \frac{\partial V}{\partial y} + \overline{a}_z \frac{\partial V}{\partial z}\right) \quad \text{... (1)}$$

Now,

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{4}{x^2 + y + z^2}\right)$$

$$= 4 \times \frac{\partial}{\partial x} (x^2 + y + z^2)^{-1}$$

$$= 4 \times (-1)\,(x^2 + y + z^2)^{(-1-1)} \times (2x + 0 + 0)$$

$$= -4\,(x^2 + y + z^2)^{-2} \times 2x$$

$$= \frac{-8x}{(x^2 + y + z^2)^2} \quad \text{... (2)}$$

$$\frac{\partial V}{\partial y} = 4 \times \frac{\partial}{\partial y}(x^2 + y + z^2)^{-1}$$

$$= 4 \times (-1)\,(x^2 + y + z^2)^{-2} \times (0 + 1 + 0)$$

$$= \frac{-4}{(x^2 + y + z^2)^2} \quad \text{... (3)}$$

$$\frac{\partial V}{\partial z} = 4 \times \frac{\partial}{\partial z}(x^2 + y + z^2)^{-1}$$

$$= 4 \times (-1)\,(x^2 + y + z^2)^{-2} \times (0 + 0 + 2z)$$

$$= \frac{-8z}{(x^2 + y + z^2)^2} \quad \text{... (4)}$$

Substituting the above three equations in equation (1), we get,

$$\therefore \quad \overline{E} = -\left(\overline{a}_x \times \frac{-8x}{(x^2 + y + z^2)^2} + \overline{a}y \times \frac{-4}{(x^2 + y + z^2)^2} + \overline{a}_z \times \frac{-8z}{(x^2 + y + z^2)^2}\right)$$

$$= \frac{4}{(x^2 + y + z^2)^2} \times (2x\,\overline{a}_x + \overline{a}_y + 2z\overline{a}_z)$$

$\therefore$ Magnitude of $\overline{E}$,

$$|\overline{E}| = \frac{4}{(x^2 + y + z^2)^2} \times \sqrt{(2x)^2 + (1)^2 + (2z)^2}$$

$$= \frac{4\sqrt{4x^2 + 4z^2 + 1}}{(x^2 + y + z^2)^2} \text{ V/m}$$

1.4 Electric Field Intensity Due to Surface Charge

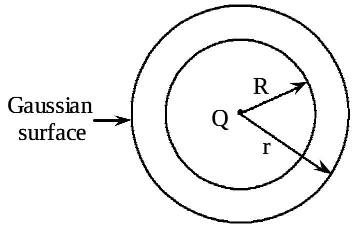
Q16. $\bar{E}$ is the electric field due to a point charge Q C at the origin in free space. Find $\int_S \bar{E}.\overline{da}$ where S is a spherical surface of radius R m and center at origin.

Feb.-08, Set-4, Q1(a) M[6]

Answer :

The Gauss's law is used to find the electric field due to a point charge 'Q' at the origin in free space.

Consider a uniformly charged sphere of radius R m and of total charge 'Q' C as shown in figure.



Figure

A spherical surface of radius 'r' is drawn such that ($r > R$) and this surface is known as 'Gaussian surface'. "According to Gauss's law, the flux through any surface enclosing the charge is $\frac{Q}{\epsilon_0}$"

Then, for closed surface 'S'

$$\oint_S E.da = \frac{1}{\epsilon_0} Q_{enc} \quad \text{... (1)}$$

Where 'da' is the infinitesimal area and it is perpendicular to field 'E'.

'E' is present inside the surface integral and its direction is radially outwards, some as 'da' which also points radially outwards, so we can cancel the dot product of 'E' and 'da'.

$$\int_S E.da = \int_S |E|\, da$$

As the magnitude of 'E' is constant over the Gaussian surface, so it comes outside the integral.

$$\int_S |E|\, da = |E| \int_S da$$

As the area of circle is πr^2 so the area of the surface S is $4\pi r^2$.

i.e., $$\int_S da = 4\pi r^2$$

$$\therefore \quad |E| \int_S da = |E| 4\pi r^2 \quad \text{... (2)}$$

Substituting equation (2) in equation (1),

Thus,

$$|E| 4\pi r^2 = \frac{1}{\epsilon_0} Q$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}}$$

Where, $\hat{r}$ is a vector which points radially outward.

As the medium is free space so, $\epsilon_r = 1$ and $\epsilon = \epsilon_0$.

Q17. A charge of Q is uniformly distributed in a half-circular ring of radius 'a'. Determine 'E' at the centre.

Answer :

Given data,

Uniformly distributed charge = Q

Radius of the half-circular ring = a

Magnitude of electric field intensity at the centre, E = ?

As, the charge is distributed uniformly, the line charge density along the length of half-circular ring is,

$$\rho_l = \frac{Q}{l}$$

Where,

l = Length of half-circular ring

$$= \frac{1}{2} \times \text{ Perimeter of circle } = \frac{1}{2} \times 2\pi r = \pi r$$

$$\therefore \quad \rho_l = \frac{Q}{\pi r}$$

Consider the EFI at centre due to a differential charge element of length dl lying on the half-circular ring as shown in the figure.

By definition of EFI, we have,

$$d\bar{E} = \frac{dQ}{4\pi\epsilon_0 R^2} \bar{a}_R$$

Where,

$$a_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-a\bar{a}_r}{a} = -\bar{a}_r$$

$$\therefore \quad d\bar{E} = \frac{(\rho_l\, d\bar{l})}{4\pi\epsilon_0 a^2} (-\bar{a}_r) \qquad [\because R = a]$$

$$= \frac{-\rho_l \times a d\phi}{4\pi\epsilon_0 a^2} \bar{a}_r \quad [\because d\bar{l} = a d\phi \text{ (from figure)}]$$

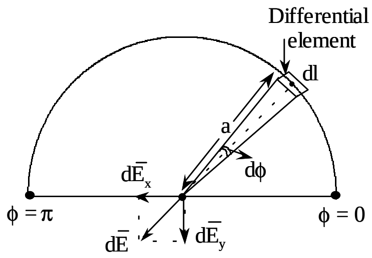


Figure: Half Circular Ring

From the figure, it is clear that by integrating $d\overline{E}$ in the interval ϕ = 0 to π radius, we get $\overline{E}$.

$$\Rightarrow \quad \overline{E} = \int_{\phi=0}^{\pi} d\overline{E}$$

$$= \int_{\phi=0}^{\pi} \frac{-\rho_l}{4\pi\epsilon_0\, a}\, d\phi\ \overline{a}_r$$

$$= \frac{-\rho_l\,\overline{a}_r}{4\pi\epsilon_0\, a}\int_{\phi=0}^{\pi} d\phi = \frac{-\rho_l\,\overline{a}_r}{4\pi\epsilon_0\, a} \times [\phi]_{\phi=0}^{\pi}$$

$$= \frac{-\rho_l\,\overline{a}_r}{4\pi\epsilon_0\, a} \times [\pi - 0] = \frac{-\rho_l\,\overline{a}_r}{4\epsilon_0\, a}$$

$$= \frac{-Q}{\pi a} \times \frac{1}{4\epsilon_0\, a}\ \overline{a}_r$$

$$= \frac{-Q}{4\pi\epsilon_0\, a^2}\ \overline{a}_r \qquad \left(\because \rho_l = \frac{Q}{\pi a}\right)$$

∴ The magnitude of electric field intensity at the centre is,

$$E = |\overline{E}| = \frac{Q}{4\pi\epsilon_0\, a^2}$$

Q18. A spherical volume charge density distribution is given by

$\rho_v = \rho_o$ (1 r^2/a^2) for r ≤ a

= 0 for r > a

Find E,

(i) Inside the charge distribution

(ii) Outside the charge distribution.

March-06, Set-2, Q2(b) M[8]

Answer :

Given data,

Volume charge density inside the sphere ($r \le a$)

$$\rho_v = \rho_o\left(1 - \frac{r^2}{a^2}\right)$$

$$\rho_v = 0$$

To calculate,

(i) Electric field intensity inside the sphere E_i = ?

(ii) Electric field intensity outside the sphere E_o = ?

(i) For r ≤ a

As the given charge distribution has spherical symmetry, consider a Guassian sphere of radius 'r' the given sphere as shown in figure (1).

According to Gauss law, the total volume charge enclosed by the surface is equal to the net flux coming out of the closed surface.

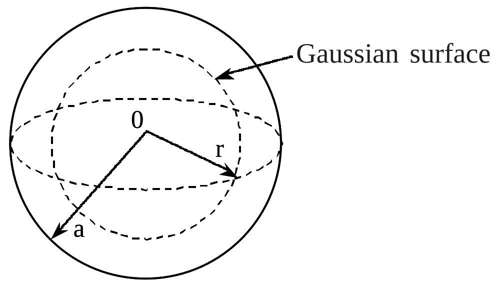


Figure (1)

$$\psi = \oint_S \overline{D}.\overline{n}ds = \text{Net charge enclosed}$$

The net volume charge enclosed

$$Q = \oint_v \rho_v dv$$

'dv' in the spherical coordinate system can be written as,

$$dv = r^2 \sin\theta\ dr\ d\theta d\phi$$

$$\therefore \quad Q = \oint_v \rho_v dv$$

$$Q = \int_0^r \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_o(1 - r^2/a^2)r^2 \sin\theta\ dr\ d\theta\ d\phi \quad \text{... (1)}$$

$$= \rho_o \int_0^r (1 - r^2/a^2)r^2\ dr \int_{\theta=0}^{\pi} \sin\theta\ d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \rho_o \int_0^r (1 - r^2/a^2)r^2\ dr \int_{\theta=0}^{\pi} [\sin\theta\ d\theta]\ 2\pi$$

$$= \rho_o \int_0^r (1 - r^2/a^2)r^2\ dr\ [-\cos\theta]_0^{\pi}\ 2\pi$$

$$= 2\pi\rho_o \int_0^r \left[1 - r^2/a^2\right] r^2 dr[-(\cos\pi - \cos 0)]$$

$$= 2\pi\rho_o \int_0^r [1 - r^2/a^2] r^2 \, dr \, [-(-2)]$$

$$= 4\pi\rho_o \int_0^r [r^2 - r^4/a^2] \, dr$$

$$= 4\pi\rho_o \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]_0^r$$

$$Q = 4\pi\rho_o \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right] \quad \ldots (2)$$

As charge enclosed, 'Q' = $\oint_S \overline{D}.\overline{n}d\overline{s}$

$$\overline{D} \oint_S d\overline{s} = 4\pi\rho_o \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]$$

$$\Rightarrow \quad \overline{D} \times 4\pi r^2 = \rho_o 4\pi \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]$$

$$\Rightarrow \quad \overline{D} = \frac{1}{r^2}\rho_o \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]$$

$$\Rightarrow \quad \overline{D} = \rho_o \left[\frac{r}{3} - \frac{r^3}{5a^2}\right]$$

$$\therefore \quad \overline{D} = \rho_o r \left[\frac{1}{3} - \frac{r^2}{5a^2}\right] a_r$$

As, $\overline{D} = \overline{E} \in_o$

$$\Rightarrow \quad \overline{E} = \frac{\overline{D}}{\in_o}$$

$$\therefore \quad \boxed{\overline{E} = \frac{\rho_o r}{\in_o}\left[\frac{1}{3} - \frac{r^2}{5a^2}\right]\overline{a}_r}$$

(ii) For r > a

Now consider a Guassian sphere of radius $r > a$ outside the given sphere as shown in figure (2).

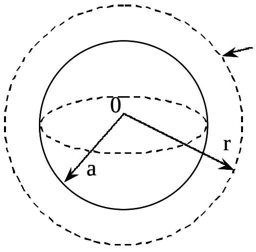


Figure (2)

For this case equation (1) can be written as,

$$Q = \int_0^a \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_o (1 - r^2/a^2) r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Then according to equation (2), we get,

$$Q = 4\pi\rho_o \left[\frac{a^3}{3} - \frac{a^5}{5a^2}\right]$$

$$Q = 4\pi\rho_o \left[\frac{a^3}{3} - \frac{a^3}{5}\right]$$

$$Q = 4\pi\rho_o \left[\frac{5a^3 - 3a^3}{15}\right]$$

$$Q = 4\pi\rho_o \left[\frac{2a^3}{15}\right]$$

$$Q = 8\pi\rho_o \, a^3/15$$

As charge enclosed, $Q = \oint_S \overline{D}.\overline{n}ds$

$$\overline{D} \oint_S d\overline{s} = 8\pi\rho_o \, a^3/15$$

$$\Rightarrow \quad \overline{D} \times 4\pi r^2 = 8\pi\rho_o \, a^3/15$$

$$\Rightarrow \quad \overline{D} r^2 = 2\rho_o \, a^3/15$$

$$\Rightarrow \quad \overline{D} = 2\rho_o \, a^3/15r^2$$

$$\therefore \quad \overline{D} = 2\rho_o \, a^3/15r^2 \; \overline{a}_r$$

As, $\overline{D} = \overline{E} \in_o$

$$\overline{E} = \frac{\overline{D}}{\in_o}$$

$$\therefore \quad \boxed{\overline{E} = \frac{2\rho_o a^3}{15r^2 \in_o}\overline{a}_r}$$

Q19. What is the value of the E field at the surface of a flat conducting sheet which has placed on it a surface charge density of $\rho_s = 10^{-2}$ C/m^2?

May-05, Set-2, Q1

Answer :

Derivation

For answer refer Unit-I, Q32.

$$\therefore \quad \overline{E} = \frac{\rho_s}{2\epsilon_0}\overline{a}_N \text{ V/m}$$

From above equation it is evident that, the electric field at any point is independent of the distance between the conducting sheet and the observation point '*P*'.

$\therefore$ $\overline{E}$ at the surface of flat conducting sheet which has placed on it a surface charge density of $\rho_s = 10^{-2}$ C/m^2 is,

$$\overline{E} = \frac{\rho_s}{2\epsilon_0}\overline{a}_N$$

$$= \frac{10^{-2}}{2\times 8.854\times 10^{-12}}\overline{a}_N$$

$$= 564.716 \times 10^6\ \overline{a}_N \text{ V/m}$$

$\therefore$ The magnitude of electric field at the surface of given flat conducting sheet is $|\overline{E}|$ = 564.716 MV/m and its direction is normal to the plane in which the flat conducting sheet is present.

Q20. The charge density inside a sphere of radius 'a' is given by $\rho = kr^2$. Find E inside and outside the sphere.

May-05, Set-4, Q1(a)

Answer :

Given data,

Radius of sphere, $r = a$

Volume charge density inside the sphere, $\rho_v = \rho = kr^2$

Electric field intensity,

Inside the sphere, E_i = ?

Outside the sphere, E_o = ?

According to Gauss law we have,

$$Q_{enc} = \oint \overline{D}.d\overline{s} = \int_v \pi dv \quad \ldots (1)$$

We know that,

$$\overline{D} = \epsilon_0 \overline{E} \text{ and}$$

$$\int_v dv = \int_{r=0}^{r}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^3 \sin\theta\, d\theta\, dr$$

$$\therefore \oint \overline{D}.d\overline{s} = \oint \epsilon_0 \overline{E}.d\overline{s}$$

$$= \epsilon_0 \overline{E}.\oint d\overline{s}$$

$$= \epsilon_0 \overline{E}.(4\pi r^2 \overline{a}_r)$$

$$= 4\pi r \epsilon_0 r^2 \overline{E}.\overline{a}_r$$

$$= 4\pi\epsilon_0 r^2(E_r \overline{a}_r + E_\phi \overline{a}_\phi + E_\theta \overline{a}_\theta).\overline{a}_r$$

$$= 4\pi\epsilon_0 r^2[E_r(1) + E_\phi(0) + E_\theta(0)]$$

$$= 4\pi\epsilon_0 r^2 E_r \quad \ldots (2)$$

$$Q = \int_v \rho dv \int_{r=0}^{r}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \rho r^2 \sin\theta\, d\phi\, d\theta\, dr$$

$$= \int_{r=0}^{r}\left[\rho r^2 \int_{\theta=0}^{\pi}\left(\sin\theta \int_{\phi=0}^{2\pi} d\phi\right) d\theta\right] dr$$

$$= \int_{r=0}^{r}\left[\rho r^2 \int_{\theta=0}^{\pi} \sin\theta [\phi]_{\phi=0}^{2\pi}\ d\theta\right] dr$$

$$= \int_{r=0}^{r}\left[\rho r^2 \int_{\theta=0}^{\pi} \sin\theta \times (2\pi - 0) d\theta\right] dr$$

$$= 2\pi \int_{r=0}^{r} \rho r^2 [-\cos\theta]_{\theta=0}^{\pi}\ dr$$

$$= 2\pi \int_{r=0}^{r} \rho r^2 [-(-1)+1] dr$$

$$= 4\pi \int_{r=0}^{r} \rho r^2\, dr \quad \ldots (3)$$

Case 1

E inside the sphere (i.e., for $r < a$)

For $r < a$, equation (3) can be written as,

$$Q_{enc} = 4\pi \int_{r=0}^{r} \rho r^2\, dr$$

$$= 4\pi \int_{r=0}^{r} kr^2.r^2\, dr = 4\pi k \int_{r=0}^{r} r^4 dr$$

$$= 4\pi k\left[\frac{r^5}{5}\right]_{r=0}^{r}$$

$$= \frac{4\pi}{5}kr^5 \quad \ldots (4)$$

Substituting equations (2) and (4) in equation (1),

$$\Rightarrow \quad 4\pi \in_0 r^2 E_r = \frac{4\pi}{5} kr^5$$

$$\Rightarrow \quad E_r = \frac{kr^3}{5\epsilon_0}$$

∴ Electric field intensity inside the sphere,

$$\boxed{E_i = \frac{kr^3}{5\in_0}} \quad \ldots (5)$$

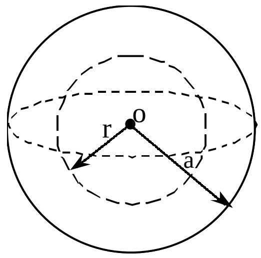


Figure (1): r < a

Case 2

E outside the sphere (i.e., for $r > a$)

For $r > a$, equation (3) can be written as,

$$Q_{enc} = 4\pi \int_{r=0}^{r} \rho r^2\, dr$$

$$= 4\pi \left[\int_{r=0}^{a} \rho r^2 dr + \int_{r=a}^{r} \rho r^2 dr \right]$$

As charge density is defined only within the sphere, so outside the sphere it will be zero.

$$\therefore \int_{r=a}^{r} \rho r^2 dv = \int_{r=a}^{r} 0.r^2 dv$$

$$= 0$$

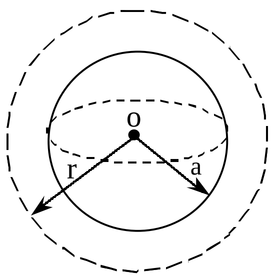


Figure (2): r > a

$$\therefore \quad Q_{enc} = 4\pi \left[\int_{r=0}^{a} \rho r^2 dr + 0 \right]$$

$$= 4\pi \int_{r=0}^{a} kr^2 . r^2 dr$$

$$= 4\pi k \left[\frac{r^5}{5} \right]_{r=0}^{a}$$

$$= \frac{4\pi}{5} ka^5 \quad \ldots (6)$$

Substituting equations (2) and (6) in equation (1),

$$\Rightarrow \quad 4\pi\epsilon_0 r^2 E_r = \frac{4\pi}{5} ka^5$$

$$\Rightarrow \quad E_r = \frac{ka^5}{5\epsilon_0 r^2}$$

∴ The electric field intensity outside the sphere,

$$\boxed{E_o = \frac{ka^5}{5\in_0 r^2}} \quad \ldots (7)$$

The electric field intensity on the surface of the sphere (i.e., at $r = a$) can be obtained from either equation (5) or from equation (7). The result obtained in either of the cases will be same as,

$$E_{(r=a)} = \frac{ka^3}{5\epsilon_0} \qquad r \leq a$$

$$= \frac{ka^5}{5\epsilon_0 r^2} \qquad r \geq a$$

Q21. Find the electric field at any point between two concentric spherical shells, inner spherical shell has Q1 charge and outer spherical shell has Q2 charge.

Nov.-04, Set-1, Q2(a)

Answer :

Consider the two concentric shells having radii R_1 and R_2 respectively. The charges Q_1 and Q_2 are uniformly distributed over the shells of radii R_1 and R_2 respectively. Here, $R_2 > R_1$ as shown in the figure.

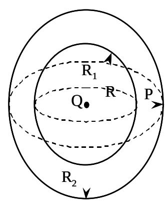


Figure: Concentric Spherical Shells

Let the medium be air and the thickness of shells is negligibly small compared to radii R_1 and R_2. The electric field intensity is assumed to be constant and normal to the surfaces.

Now, from Gauss's law, we have,

$$\in_0 \iint_s \overline{E} \cdot d\overline{s} = \text{Charge enclosed by the surface.}$$

The point 'P' between the two concentric shell may lie on the surface of inner radius (i.e., on R_1) or in between R_1 and R_2 or on the surface of R_2. If r is considered as the distance of point 'P' from the centre '0'. Let us analyze all the three cases.

Case 1

The point 'P' lies on the surface of inner shell. i.e., $r = R_1$, then,

According to Gauss's law,

$$\in_0 \iint_s \overline{E} \cdot d\overline{s} = \text{Total charge enclosed}$$

$$\in_0 \overline{E}_r.(4\pi R_1^2 \overline{a}_r) = Q_1$$

$$\Rightarrow \quad \overline{E}_r.\overline{a}_r = \frac{Q_1}{4\pi \in_0 R_1^2}$$

$$\Rightarrow \quad E_r = \frac{Q_1}{4\pi \in_0 R_1^2} \overline{a}_r \ \text{V/m}$$

$$\therefore \quad \overline{E} = \frac{Q_1}{4\pi \in_0 R_1^2} \overline{a}_r \ \text{V/m}$$

Case 2

If the point 'P' lies in between the radii R_1 and R_2

$R_1 < r < R_2$.

Then, according to the Gauss's law, we have,

$$\in_0 \iint_s \overline{E} \cdot d\overline{s} = \text{Total charge enclosed}$$

$$\Rightarrow \quad \in_0 \overline{E}.(4\pi r^2 \overline{a}_r) = Q_1$$

$$\Rightarrow \quad \overline{E}_r = \frac{Q_1}{4\pi \in_0 r^2} \overline{a}_r \ \text{V/m}$$

$$\therefore \quad \overline{E} = \frac{Q_1}{4\pi \in_0 r^2} \overline{a}_r \ \text{V/m}$$

Case 3

If the point 'P' lies on the circumference of shell having radii R_2 i.e., $r = R_2$. Then,

According to the Gauss's law, we have,

$$\in_0 \iint_s \overline{E} \cdot d\overline{s} = \text{Total charge enclosed}$$

$$\Rightarrow \quad \in_0 \overline{E}.(4\pi R_2^2 \overline{a}_r) = Q_1 + Q_2$$ [$\therefore$ The radii R_2 encloses both the charges]

$$\Rightarrow \quad E_r = \frac{(Q_1 + Q_2)}{4\pi \in_0 R_2^2} \overline{a}_r$$

$$\therefore \quad \overline{E} = \frac{(Q_1 + Q_2)}{4\pi \in_0 r^2} \overline{a}_r \ \text{V/m}$$

Q22. A circular disc of 10 cm radius is charged uniformly with a total charge of 100 μC. Find E at a point 20 cm on its axis.

Nov.-04, Set-2, Q1(b)

Answer :

Given data,

Radius, r = 10 cm

Charge, Q = 100 μC

Distance of point from origin, y = 20 cm

Consider a circular disc of radius 10 cm in the coordinate axes as shown in figure (1). Let y cm be the distance of the paint A from origin. Let dA be a differential area of the disc 10. Let Δ be the distance between the point A and the differential element ds.

The equation of dE is given by,

$$dE = \frac{dQ}{4\pi\epsilon_0 \Delta^2} \bar{a}_\Delta \quad \ldots (1)$$

The total electric field over the surface area 'S' is given by,

$$E = \int_S \frac{dQ}{4\pi\epsilon_0 \Delta^2} \bar{a}_\Delta \quad \ldots (2)$$

In cylindrical coordinate system, ds is given by,

$$dA = r\, dr\, d\phi$$

$$\bar{a}_\Delta = -r\,\bar{a}_r + y\,\bar{a}_y$$

$$\bar{a}_\Delta = \frac{-r\,\bar{a}_r + y\,\bar{a}_y}{\sqrt{r^2 + y^2}}$$

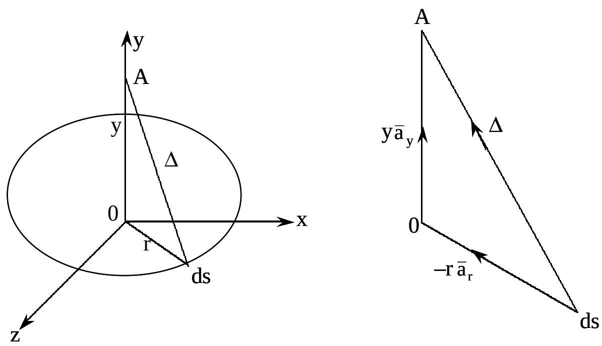


Figure (1) **Figure (2)**

From figure (2), $\Delta^2 = r^2 + y^2$

Substituting the above equation in equations (2), we get,

$$\therefore \quad E = \int_{\phi=0}^{2\pi} \int_{r=0}^{0.1} \frac{dQ}{4\pi\epsilon_0 (r^2 + y^2)} \frac{y\,\bar{a}_y}{\sqrt{r^2 + y^2}} \quad \ldots (3)$$

Let ρ be the surface charge density in Coulomb/m^2

$$\rho = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{A} \quad \ldots (4)$$

Area of the disc $= \pi r^2$

$= 3.14 \times (0.1 \times 0.1)$

$= 0.0314 \text{ m}^2$

$$\rho = \frac{100\times10^{-6}}{0.0314}$$

$= 3.184 \times 10^{-3} \text{ C/m}^2$

∴ From equation (4),

$Q = \rho A$

$dQ = \rho dA$ ... (5)

Substituting equation (5) in equation (3), we get,

$$E = \int_{\phi=0}^{2\pi}\int_{r=0}^{0.1} \frac{\rho[rdr\,d\phi]}{4\pi\epsilon_0 (r^2+y^2)} \frac{y\bar{a}_y}{\sqrt{r^2+y^2}}$$

$$E = \frac{\rho y}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi}\int_{r=0}^{0.1}\frac{rdr\,d\phi}{(r^2+y^2)^{1.5}}\bar{a}_y$$

Let, $r^2 + y^2 = P^2$ and $rdr = PdP$

The limits of equation (6) changes as,

$r = 0, P_1 = y, r = 0.1, P^2 = \sqrt{0.1^2 + y^2}$

Substituting the above values in equation (6), we get,

$$E = \frac{\rho y}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi}\int_{P_1}^{P_2}\frac{PdPd\phi}{P^3}\bar{a}_y$$

$$E = \frac{\rho y}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi}d\phi\int_{P_1}^{P_2}\frac{PdP}{P^3}\bar{a}_y$$

$$E = \frac{\rho y}{4\pi\epsilon_0}[\phi]_0^{2\pi}\int_{P_1}^{P_2}\frac{dP}{P^2}\bar{a}_y$$

$$E = \frac{\rho y\times 2\pi}{4\pi\epsilon_0}\left[-\frac{1}{P}\right]_{P_1}^{P_2}\bar{a}_y$$

$$E = \frac{\rho_y}{2\epsilon_0}\left[\frac{1}{P_1}-\frac{1}{P_2}\right]\bar{a}_y \quad \text{... (7)}$$

We know that, $\rho = 3.184 \times 10^{-3}$, $y = 20$ cm $= 0.2$ m

$P_1 = 0.2$ m, $P_2 = \sqrt{(0.1)^2+(0.2)^2} = \sqrt{0.05} = 0.2236$

Substituting the above values in equation (7), we get,

$$E = \frac{3.184\times10^{-3}\times0.2}{2\times8.854\times10^{-12}}\left[\frac{1}{0.2}-\frac{1}{0.2236}\right]$$

∴ $E = 18.97 \times 10^6$ V/m

Q23. Point charges are located at each corner of an equilateral triangle. If the charges are 3Q, –2Q and 1Q, find electric field at midpoint of 3Q and 1Q side.

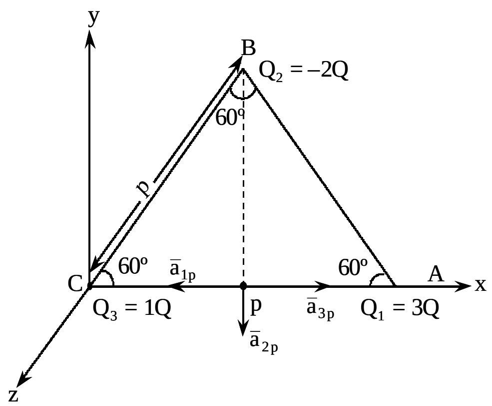
Answer :

Given that,

The three point charges of charge $3Q$, $-2Q$ and $1Q$ are at corners of an equilateral triangle.

Electric field at midpoint of $3Q$ and $1Q$ side, $\bar{E} = ?$

Let the given point charges $3Q$, $-2Q$ and $1Q$ be Q_1, Q_2 and Q_3 at points A, B and C respectively, in xy plane with side CA lying parallel to x-axis as shown in the figure.



Also, let the midpoint of side CA be 'p' and the length of each side be 'a'.

We know that, the electric field intensity at a point 'p' due to a point charge distribution is,

$$\bar{E}_p = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{Q_i}{(R_{ip})^2}\bar{a}_{ip}$$

Where, n = Number of charges = 3

$$\bar{E}_p = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{3}\frac{Q_i}{(R_{ip})^2}\bar{a}_{ip}$$

$$\bar{E}_p = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1}{R_{1p}^2}\bar{a}_{1p}+\frac{Q_2}{R_{2p}^2}\bar{a}_{2p}+\frac{Q_3}{R_{3p}^2}\bar{a}_{3p}\right]$$

From the figure, we have,

$$R_{1p} = \overline{AP} = \frac{\overline{AC}}{2} = \frac{a}{2}$$

$$\bar{a}_{1p} = -\bar{a}_x.$$

$$R_{2p} = \overline{BP} = \overline{BC} \times \sin 60^{\circ}$$

$$= a \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}a}{2}$$

$$\bar{a}_{2p} = -\bar{a}_y$$

$$\bar{R}_{3p} = \overline{CP} = \frac{\overline{AC}}{2} = \frac{a}{2}$$

$$\bar{a}_{3p} = \bar{a}_x$$

$$\therefore \quad \bar{E}_p = \frac{1}{4\pi\epsilon_0}\left[\left\{\frac{3Q}{\left(\frac{a}{2}\right)^2}\times(-\bar{a}_x)\right\}+\left\{\frac{(-2Q)}{\left(\frac{\sqrt{3}a}{2}\right)^2}\times(-\bar{a}_y)\right\}+\left\{\frac{1Q}{\left(\frac{a}{2}\right)^2}\times(\bar{a}_x)\right\}\right]$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{Q}{\frac{a^2}{4}} \left[-3\bar{a}_x + \frac{2}{3}\bar{a}_y + \bar{a}_x\right]$$

$$= \frac{Q}{\pi\epsilon_0 a^2}\left[-2\ \bar{a}_x + \frac{2}{3}\bar{a}_y\right]$$

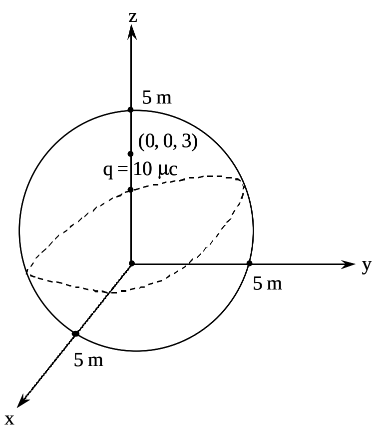
The magnitude of electric field intensity is $\frac{Q}{\pi\epsilon_0 a^2}$ and the direction is indicated by $(-2\bar{a}_x + \frac{2}{3}\bar{a}_y)$.

Q24. Find the flux of the electric field through a spherical surface of radius 5 m and center origin, in free space if there is a charge of 10 μC at the point (0, 0, 3)m. What are its units?

Nov.-07, Set-2, Q1(c) M[4]
Feb.-07, Set-2, Q1(c) M[6]
Nov.-06, Set-2, Q1(c) M[6]
Nov.-06, Set-3, Q1(c) M[6]

Answer :

Given data,



We know that according to Gauss law the total flux of electric field through the closed surface is equal to the total charge enclosed by the closed surface.

i.e., Flux, ψ = Charge (q)

= 10 μc

Units for electric flux are Coulombs.

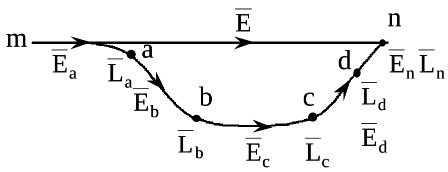
1.5 Work done in Moving a Point Charge in an Electrostatic Field

Q25. Show that the work done in moving a charge from one point to another in an electrostatic field is independent of the path between the points.

Feb.-07, Set-1, Q1(a) M[6]

Answer :

Consider a point charge of 'Q' coulombs be placed in a uniform electric field or electrostatic field, located at a point 'm'. Let, the point charge to be moved to another point 'n' in the same field along the path shown in figure. Consider the path to be in five different sections m to a (L_a), a to b (L_b), b to c (L_c), c to d (L_d), d to n (L_n). Let, the components of the electric field intensity (E) along these sections be E_a, E_b, E_c and E_n.



Figure

The total work done in moving the point charge from point 'm' to point 'n' is,

$$W = -QE_aL_a - QE_bL_b - QE_cL_c - QE_dL_d - QE_nL_n$$

$$= -Q(E_aL_a + E_bL_b + E_cL_c + E_dL_d + E_nL_n) \quad \text{... (1)}$$

Equation (1) can be rewritten in vector representation as,

$$W = -Q(\bar{E}_a.\bar{L}_a + \bar{E}_b.\bar{L}_b + \bar{E}_c.\bar{L}_c + \bar{E}_d.\bar{L}_d + \bar{E}_n.\bar{L}_n)$$

As the field is uniform, we have,

$$\bar{E}_a = \bar{E}_b = \bar{E}_c = \bar{E}_d = \bar{E}_n = \bar{E}$$

$$\Rightarrow \quad W = -Q(\bar{E}.\bar{L}_a + \bar{E}.\bar{L}_b + \bar{E}.\bar{L}_c + \bar{E}.\bar{L}_d + \bar{E}.\bar{L}_n)$$

$$= -Q\bar{E}(\bar{L}_a + \bar{L}_b + \bar{L}_c + \bar{L}_d + \bar{L}_n)$$

$$= -Q\bar{E}\,\bar{L}_{mn} \quad \text{[By parallelogram law]}$$

Where, $\bar{L}_{mn}$ = Vector $\bar{L}$ directed from point 'm' to point 'n'.

Hence, the work done in moving a charge from one point to another in an electrostatic field is independent of the path between the points.

So, the work done can be obtained simply by using the line integration as,

$$W = -Q\int_m^n \bar{E}.d\bar{L}$$

$$= -Q\bar{E}.\int_m^n d\bar{L} \quad [\because \text{Uniform field}]$$

$$= -Q\bar{E}.[L]_m^n$$

$$= -Q\bar{E}.[\bar{L}_n - \bar{L}_m]$$

$$= -Q\bar{E}.\bar{L}_{mn}$$

$$W = -Q\bar{E}.\bar{L}_{mn}$$

Q26. Define the term "potential difference V(A) – V(B), between points A and B in a static electric field". Explain the concept of reference point and comment on its location.

Nov.-07, Set-1, Q1(a) M[6]
Feb.-07, Set-4, Q1(a) M[4]
Nov.-06, Set-1, Q1(a) M[4]

Answer :

Potential Difference

Potential difference $V(A) - V(B)$, between points A and B in a static electric field is defined as the work done by an external force in moving a unit positive charge from point A to point B in an electric field.

We know that, work done in moving a charge from point A to point B,

$$W = -Q\int_A^B \overline{E}.d\overline{L}$$

$$\therefore \quad V_{AB} = V(A) - V(B) = \frac{W}{Q}$$

$$= \frac{-Q\int_A^B \overline{E}.d\overline{L}}{Q}$$

$$= -\int_A^B \overline{E}.d\overline{L}$$

$$= \int_B^A \overline{E}.d\overline{L} \text{ J/C or V.}$$

Concept of Reference Point and its Location

A reference point or zero reference is a common point which is considered to have zero potential. In most of the cases earth is taken as reference. In some applications like inter planetary mission where the earth is too far, infinity is taken as a reference point. In case of under ground cables, the outer conductor which surrounds the inner conductor is taken as zero reference from this it can be seen that the reference point is not always far away from the point of consideration and the selection of its location purely depends up on the convenience point of view.

Q27. Calculate the work done in moving a point charge of 10 μC from point (4, 90º, 60º) to (3, 30º, 120º) if V = $\frac{10}{r}$ cosθ sinϕ.

Answer :

Given data,

Point charge, $Q = 10\ \mu C$

$= 10 \times 10^{-6}$ C

Point charge is moved from point $P_1 = (4, 90^o, 60^o)$ to point $P_2 = (3, 30^o, 120^o)$

Potential, $V = \frac{10}{r}\cos\theta\sin\phi$

Work done, W = ?

We know that, the work done in moving a point charge from point A to point B is,

$$W = -\int_A^B \overline{E}.d\overline{L} = -\int_A^B Q\overline{E}.d\overline{L}$$

$$= -Q\int_A^B \overline{E}.d\overline{L}$$

$$= Q[V_B - V_A]$$

$$\left[\because V_{AB} = V_B - V_A = -\int_A^B \overline{E}.d\overline{L}\right]$$

$$= Q[V_{P_2} - V_{P_1}] \quad \text{... (1)}$$

[$\because$ Here A and B are P_1 and P_2 respectively]

Potential at point P_1,

$$V_{P_1} = \frac{10}{r}\cos\theta\sin\phi\Big|_{(4,90°,60°)}$$

$$= \frac{10}{4}\cos 90^o \sin 60^o = 0 \text{ V}$$

Potential at point P_2,

$$V_{P_2} = \frac{10}{r}\cos\theta\sin\phi\Big|_{(3,30°,120°)}$$

$$= \frac{10}{3}\cos 30° \sin 120° = 2.5$$

$\therefore$ Work done, $W = Q[V_B - V_A]$

$= 10 \times 10^{-6}[2.5 - 0]$

$= 25 \times 10^{-6}$ J = 25 μJ

1.6 Electric Potential

Q28. (i) Find the potential at a point external to a spherical surface of uniform surface charge density and radius R m in free space. Use the principle of superposition potentials.

(ii) Repeat (i) If the point is internal to the spherical surface.

(iii) Comment on the results.

Feb.-07, Set-1, Q1(b) M[6+2+2]

Answer :

Given data,

Spherical surface has uniform surface charge density,

Radius of the sphere = R m

Sphere is in free space

Use principle of superposition potentials,

(i) Potential at a point external to the spherical surface, $V_{ext} = ?$

(ii) Potential at a point internal to the spherical surface, $V_{int} = ?$

We know that the potential at a point 'p' due to a point charge 'Q' C is,

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Where, 'r' is the distance between point charge and point 'p'.

According to the principle of superposition potential we have, potential at point 'p' due to 'n' point charges is,

$$V = \frac{Q_1}{4\pi\epsilon_0 r_1} + \frac{Q_2}{4\pi\epsilon_0 r_2} + ... + \frac{Q_n}{4\pi\epsilon_0 r_n}$$

$$= \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{n}\frac{Q_k}{r_k}$$

For continuous charge distributions the point charge Q_k has to be replaced with the corresponding charge elements (i.e., $\rho_L dL$ or $\rho_S\, ds$ or $\rho_V\, dv$) and the summation has to be replaced with corresponding integration (i.e., line or surface or volume).

∴ For surface charge distribution,

$$V = \frac{1}{4\pi\epsilon_0}\iint_s \frac{\rho_S\, ds}{r} \quad ... (1)$$

(i) Let, the surface charge density be ρ_S C/m^2, the distance from the center of the spherical surface to the point external to it be r_m.

∴ Potential at a point external to the spherical surface,

$$V_{ext} = \frac{1}{4\pi\epsilon_0}\iint_s \frac{\rho_S\, ds}{r_m}$$

$$= \frac{\rho_S}{4\pi\epsilon_0 r_m}\iint_s ds \qquad [\because \rho_S \text{ is uniform throughout the spherical surface}]$$

$$= \frac{\rho_S}{4\pi\epsilon_0 r_m} \times 4\pi R^2$$

$$\therefore \quad V_{ext} = \frac{\rho_S R^2}{\epsilon_0 r_m}\text{V} \quad ... (2)$$

(ii) The potential on the spherical surface is,

$$V_{int} = \frac{\rho_S}{4\pi\epsilon_0 R}\iint ds \quad ... (3)$$

$$V_{int} = \frac{\rho_S 4\pi R^2}{4\pi\epsilon_0 R}$$

$$V_{int} = \frac{\rho_S R}{\epsilon_0}\text{V} \quad ... (4)$$

If 'r' is distance from center of spherical surface to the point 'p' internal to it, i.e., $r \le R$ then potential at any point $r \le R$ is, given by,

V = Potential on the surface + Work involved in moving a test charge from R to r inside

$$\therefore \quad V = \frac{\rho_S}{4\pi\epsilon_0 R}\iint ds - \int_R^r \overline{E}.d\overline{r}$$

$$\therefore \quad \boxed{V = \frac{\rho_S R}{\epsilon_0} - \int_R^r \overline{E}.d\overline{r}}$$

(iii) The above results show that V_{ext} is always greater than V_{int}.

Q29. Find the value of electric potential at the point at which E = 0 when point charge of 3 μC and 5 μC are located at (0.0, 0) and (0.6, 0) m in XY plane.

March-06, Set-2, Q1(b) M[8]

Answer :

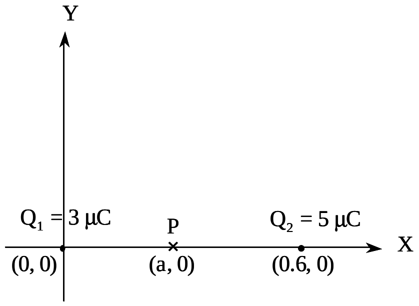
Given data,

Point charge at (0, 0), $Q_1 = 3\ \mu C$

Point charge at (0.6, 0), $Q_2 = 5\ \mu C$

Point, P = ? where $E = 0$

At P, electric potential, V = ?



Figure

Since both the charges Q_1 and Q_2 lie on X-axis only the point 'P' where the electric field intensity becomes zero will also lie on X-axis in between Q_1 and Q_2.

So, let $P = (a, 0)$

Electric field intensity at 'P' due to $Q_1(E_1)$ = Electric field intensity at 'P' due to $Q_2(E_2)$.

i.e., $E_1 = E_2$

$$\Rightarrow \quad \frac{Q_1}{4\pi\epsilon_o r_1^2} = \frac{Q_2}{4\pi\epsilon_o r_2^2}$$

$$\Rightarrow \quad \frac{Q_1}{r_1^2} = \frac{Q_2}{r_2^2}$$

$$\Rightarrow \quad \frac{3\times10^{-6}}{a^2} = \frac{5\times10^{-6}}{(a-0.6)^2}$$

$\Rightarrow \quad 3(a^2 - 1.2a + 0.36) = 5a^2$

$\Rightarrow \quad 5a^2 - 3a^2 + 3.6\,a - 1.08 = 0$

$\Rightarrow \quad 2a^2 + 3.6\,a - 1.08 = 0$

$\Rightarrow \quad a = 0.2619$ or -2.0619

As the point 'P' must lie in between the charges,

$\therefore \quad a = 0.2619$

i.e., $\quad P = (0.2619, 0)$

$\therefore$ Potential at point P,

$$V = \frac{Q_1}{4\pi\epsilon_o r_1^2} + \frac{Q_2}{4\pi\epsilon_o r_2^2}$$

$$= \frac{1}{4\pi\epsilon_o}\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right)$$

$$= \frac{1}{4\pi\times8.854\times10^{-12}}\times\left(\frac{3\times10^{-6}}{(0.2619)^2} + \frac{5\times10^{-6}}{(0.6-0.2619)^2}\right)$$

= 786206.015 V

$\simeq$ 786.206 kV.

Q30. An infinitely large cylinder has a radius and a uniform charge of one micro coulomb per meter. Calculate the potential at a point 10 m away from the cylinder if zero potential point is taken to be at a radial distance of 1 m.

Nov.-05, Set-2, Q1 M[16]

Nov.-05, Set-3, Q1 M[16]

Answer :

Given data,

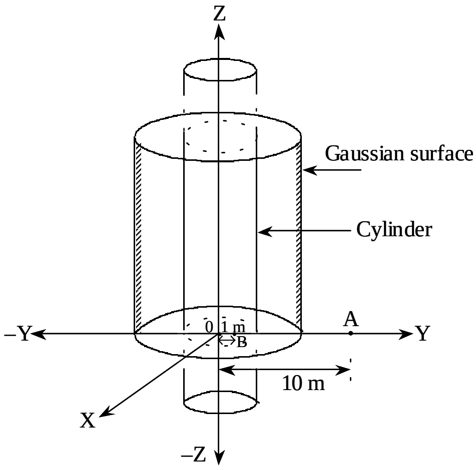
Charge, $Q = 1$ μ Coulomb $= 1 \times 10^{-6}$ C

Potential at radial distance of 1 m = 0 V

Potential at a point 10 m = ?

Derivation

Assume that the zero potential occurs at point B and the potential at point A is to be calculated such that A is 10 m away from B, as shown in the figure.



Figure

Assume a Gaussian surface around the cylinder as shown in the figure. Since, the charge uniformly distributed throughout the cylinder, the cylinder can be assumed as an infinite line charge.

According to the Gauss's law, total flux emanating (coming out) from a closed surface area is equal to the charge enclosed by that surface area.

i.e., $$Q = \oint_s \overline{D}.\overline{ds} \quad \dots (1)$$

Here, the imaginary cylinder has three areas namely top, bottom and curved surface area. Consequently, the total flux emanating is equal to the sum of the fluxes emanating (coming out) from these three areas. Hence, equation (1) can be written as,

$$Q = \oint_{\text{curved surface}} \overline{D}.\overline{ds} + \oint_{\text{top}} \overline{D}.\overline{ds} + \oint_{\text{bottom}} \overline{D}.\overline{ds}$$

$$\Rightarrow \quad Q = \oint_{\text{curved surface}} \in_0 \overline{E}.\overline{ds} + \oint_{\text{top}} \in_0 \overline{E}.\overline{ds} + \oint_{\text{bottom}} \in_0 \overline{E}.\overline{ds}$$

$[\because \overline{D} = \in_0 \overline{E}$ for free space$]$

$$\Rightarrow \quad Q = \in_0 \overline{E}\left[\oint_{\text{curved surface}} \overline{ds} + \oint_{\text{top}} \overline{ds} + \oint_{\text{bottom}} \overline{ds}\right]$$

$$\Rightarrow \quad Q = \in_0 \overline{E}\,[2\pi rl\,\overline{a}_r + 2\pi r\,\overline{a}_z - 2\pi r\,\overline{a}_z]$$

$$\therefore \quad Q = 2\pi rl \in_0 \overline{E}\,.\,\overline{a}_r \quad \dots (2)$$

In cylindrical coordinates, $\overline{E}$ can be expressed as,

$$\overline{E} = E_r\overline{a_r} + E_\phi\,\overline{a_\phi} + E_Z\overline{a_Z}$$

As electric field intensity ($\overline{E}$) acts radial to the Gaussian surface, only 'r' components exists.

$$\therefore \quad \overline{E} = E_r\overline{a}_r \quad \dots (3)$$

Substituting equation (3) in equation (2), we get,

$$Q = 2\pi rl \in_0 E_r\overline{a}_r\,.\,\overline{a}_r$$

$$Q = 2\pi rl \in_0 E_r.1$$

$$Q = 2\pi rl \in_0 E_r \quad \dots (4)$$

But, the total charge is given by,

$$Q = \rho_l \times l$$

$$\therefore \quad \rho_l \times l = 2\pi rl \in_0 E_r$$

$$E_r = \frac{\rho_l}{2\pi\in_0 r} \quad \dots (5)$$

Substituting equation (5) in equation (3), we get,

$$\overline{E} = \frac{\rho_l}{2\pi\in_0 r}\overline{a}_r \text{ V/m}$$

Calculations

Potential difference between the points A and B is given by,

$$V_{AB} = -\int_B^A \overline{E}.\overline{dr}$$

$$V_{AB} = -\int_B^A \frac{\rho_l}{2\pi\in_0 r}.\overline{a}_r(.dr.\overline{a}_r)$$

$$V_{AB} = -\int_B^A \frac{\rho_l}{2\pi\epsilon_0 r}.dr \qquad [\because \bar{a}_r.\bar{a}_r = 1]$$

$$V_{AB} = -\frac{\rho_l}{2\pi\epsilon_0}\int_B^A \frac{1}{r}.dr$$

$$V_{AB} = -\frac{\rho_l}{2\pi\epsilon_0}[\log r]_B^A$$

$$V_{AB} = \frac{-\rho_l}{2\pi\epsilon_0}[\log A - \log B]$$

$$V_{AB} = \frac{\rho_l}{2\pi\epsilon_0}[\log B - \log A]$$

$$V_{AB} = \frac{\rho_l}{2\pi\epsilon_0}\log\left(\frac{B}{A}\right)$$

$$V_{AB} = \frac{\rho_l}{2\pi\epsilon_0}\log\left(\frac{1}{10}\right)$$

$$V_{AB} = \frac{-\rho_l}{2\pi\epsilon_0}\log 10$$

$$V_{AB} = \frac{-1\times10^{-6}}{2\pi\times8.854\times10^{-12}} \times 1$$

$$V_{AB} = -17.975 \text{ kV}$$

The negative sign signifies that the energy is expended by the electric field of infinitely large cylinder in moving the positive charge from point A to point B.

$\therefore$ The potential at a point 10 m away from the cylinder is – 17.975 kV, when a zero potential point is taken at a radial distance of 1 m.

Q31. What are the equipotential for an infinite straight line of uniform charge density? Explain.

Feb.-08, Set-1, Q1(b) M[6]
Nov.-07, Set-1, Q1(b) M[6]
Nov.-07, Set-3, Q1(b) M[6]
Feb.-07, Set-4, Q1(b) M[6]
Nov.-06, Set-1, Q1(b) M[6]

Answer :

The electric field intensity at a point 'P' due to an infinite straight line of uniform charge density (ρ_L) is,

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 r}\bar{a}_r \text{ V/m}$$

Where, r = Radial distance from point P to infinite straight line.

Derivation

For answer refer Unit-I, Q30, Topic: Derivation.

Consider points A and B located at a distance of r_a and r_b respectively from infinite straight line.

Potential between points A and B is,

$$V_{AB} = \int_{r_a}^{r_b} \bar{E}.d\bar{L} = -\int_{r_a}^{r_b} \frac{\rho_L}{2\pi\epsilon_0 r} dr\,\bar{a}_r.\bar{a}_r$$

$$= \frac{-\rho_L}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{1}{r}dr$$

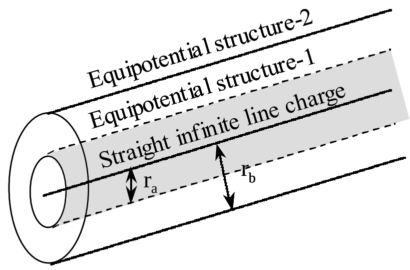
$$= \frac{-\rho_L}{2\pi\epsilon_0}[\ln r]_{r_a}^{r_b}$$

$$= \frac{-\rho_L}{2\pi\epsilon_0}[\ln r_b - \ln r_a]$$

$$= \frac{\rho_L}{2\pi\epsilon_0}[\ln r_a - \ln r_b]$$

$$= \frac{\rho_L}{2\pi\epsilon_0}\ln\left(\frac{r_a}{r_b}\right) \text{V}$$

Hence, for all the points which are at a distance of r_b will have a potential difference of $\frac{\rho_L}{2\pi\epsilon_0}\ln\left(\frac{r_a}{r_b}\right)$ V with that of the points at a distance of r_a. So, the entire cylindrical structure formed around the infinite straight line charge will form an equipotential structure i.e., the equipotential for an infinite line of uniform charge density are the cylinder structures of infinite length as shown in the figure.



Figure

Q32. What are the equipotential surfaces for an infinite plane of uniform surface charge density? Explain.

Feb.-07, Set-3, Q1(b) M[6]

Answer :

The electric field intensity at any point due to infinite plane of uniform charge density (ρ_s) is given by,

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\bar{a}_n \text{ V/m}$$

Electric Field at any Point Due to Infinite Charge Surface using Gauss Law

Consider an infinite charged surface of charge density 'ρ_s' as shown in the figure (1) and placed in the *YZ* plane.

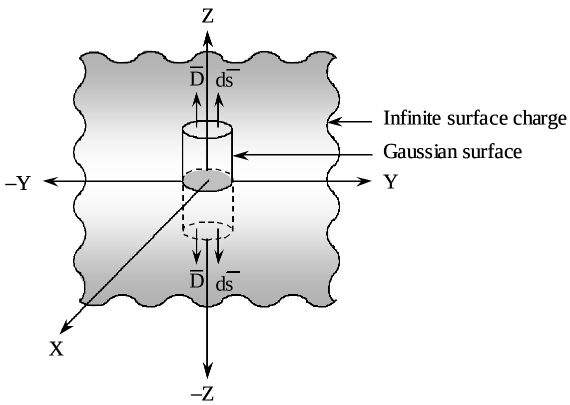


Figure (1): Infinite Disc

Consider a small cylindrical Gaussian surface placed on infinite charge surface such that it encloses a small area of the surface, denoted by '$\bar{ds}$'.

Let the flux coming out of the cylindrical Gaussian surface be '$d\phi$' and 'dQ' denotes the charge enclosed by the cylinder. Applying Gauss law to the cylinder, we get,

$$dQ = d\phi$$

$$dQ = \bar{D}.\bar{ds}\,|_{\text{curved surface}} + \bar{D}.\bar{ds}\,|_{\text{top}} + \bar{D}.\bar{ds}\,|_{\text{bottom}}$$

But, the flux coming out of the curved surface is zero. Therefore,

$$dQ = 0 + \bar{D}.\bar{ds} + \bar{D}.\bar{ds}$$

$$dQ = 2\bar{D}.\bar{ds}$$

$$\rho_s ds = 2\bar{D}.\bar{ds} \quad [\because Q = \rho_s \times s \Rightarrow dQ = \rho_s ds]$$

$$\bar{D} = \frac{\rho_s}{2}\bar{a}_z \quad [\because \bar{D} \text{ is normal to the infinite charge surface } \bar{D} = D\bar{a}_z] \quad \text{... (1)}$$

But, we know that,

$$\bar{D} = \epsilon_0 \bar{E} \quad \text{... (2)}$$

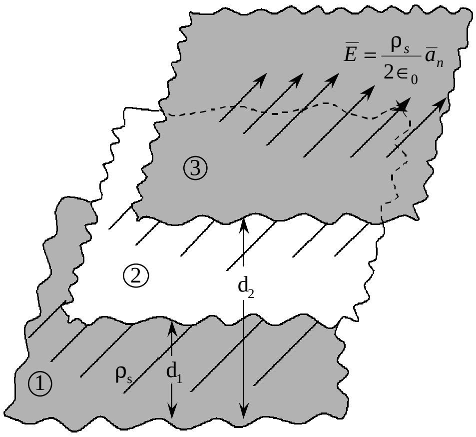
Using equation (2) in equation (1), we get,

$$\epsilon_0 \bar{E} = \frac{\rho_s}{2}\bar{a}_z$$

$$\Rightarrow \quad \bar{E} = \frac{\rho_s}{2\epsilon_0}\bar{a}_z \text{ V/m}$$

In general, the electric field intensity at any point due to infinite charge surface is given by,

$$\bar{E} = \frac{\rho_s}{2\epsilon_0}\bar{a}_n \text{ V/m.}$$



① Infinite plane of uniform surface charge density.

② Infinite plane formed by the equipotential surface at a distance d_1 from (1)

③ Infinite plane formed by the equipotential surface at a distance d_2 from (1)

Figure (2)

Potential at any point '*p*' which is located at a distance '*d*' from the infinite plane,

$$V = \frac{\rho_s}{2\epsilon_0}\times d$$

So, the entire infinite plane which is at a distance of *d* mts (normally) from the infinite plane of uniform charge density (1) will have the same potential of,

$$V = \frac{\rho_s d}{2\epsilon_0}$$

This entire plane is said to be an equipotential surface for the infinite plane of uniform surface charge density, as the potential at all the points in this plane are equal. Two such equipotential surfaces (2) and (3) are shown in the figure, which are at a distance of d_2 and d_3 from (1) respectively.

Q33. A uniformly charged spherical surface of radius 0.5 m is in free space. If the potential at the surface is 100 V (reference at infinity) what is the surface charge density?

Nov.-07, Set-1, Q1(c) M[4]
Feb.-07, Set-4, Q1(c) M[6]

Answer :

Given data,

If *Q* is the uniformly distributed charge over the spherical surface then electric field intensity,

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r \quad (r \geq R)$$

∴ Absolute potential at the surface of the spherical shell,

$$V_0 = -\int_{\infty}^{R} \bar{E}\cdot d\bar{L}$$

$$\Rightarrow V_0 = -\int_{\infty}^{R} \frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r\ (dr\,\bar{a}_r)$$

$$\Rightarrow V_0 = -\int_{\infty}^{R} \frac{Q}{4\pi\epsilon_0 r^2}dr$$

$$\Rightarrow V_0 = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{R}\frac{1}{r^2}dr$$

$$\Rightarrow V_0 = -\frac{Q}{4\pi\epsilon_0}\left[\frac{r^{-2+1}}{-2+1}\right]_{\infty}^{R}$$

$$\Rightarrow V_0 = -\frac{Q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{R}$$

$$\Rightarrow V_0 = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R}-0\right]$$

$$\Rightarrow V_0 = \frac{Q}{4\pi\epsilon_0 R}$$

$\Rightarrow Q = 4\pi\epsilon_0 R \times V_0$

$\Rightarrow Q = 4\pi \times 8.854 \times 10^{-12} \times 0.5 \times 100$

$\Rightarrow Q = 5.563 \times 10^{-9}$ C

$\therefore Q = 5.563$ nC

∴ Surface charge density,

$$\rho_S = \frac{Q}{\text{Surface area of spherical shell}}$$

$$= \frac{Q}{4\pi R^2} = \frac{5.563\times10^{-9}}{4\pi\times(0.5)^2}$$

$$\therefore \rho_S = 1.771\ \text{nC/m}^2$$

1.7 Properties of Potential Function

Q34. What is a potential function? List out the properties of a potential function.

Answer :

Potential Function (V)

Potential function '*V*' is not uniquely defined. Any quantity which is independent of the co-ordinates can be added to it without effecting the electric field in any way.

"The electrostatic field 'E' can be described completely by means of a potential function 'V'(x, y, z), which is known as electric potential". Thus,

$$E = -\Delta V$$

Where, 'V' is a scalar point function.

Electric field intensity is the derivative of potential function i.e., $\dfrac{dV}{dx} = -Eax$

It means that if the two points have non-zero potential difference, then only the electric lines of forces should be present. These lines of forces travel from higher potential conductor to lower potential conductor.

Properties of Potential Function

Potential function has different properties as follows,

1. Potential function has a single value at any point in any electrostatic field.
2. Potential function is a continuous function.
3. The difference in potential between any two points does not depends on the path of integration.
4. An electrostatic field is conservative in nature since $\Delta \times \Delta V = 0$ that is the work done in moving a point charge around any closed path in the field is zero.

Q35. $\Delta V = xa_x + ya_y + za_z$. If (1, 1, 1) m is at zero volts, find the potential V(x,y,z).

Nov.-06, Set-4, Q1(c) M[4]

Answer :

Given data,

$\Delta V = xa_x + ya_y + za_z$

The point (1, 1, 1) m is at zero volts.

Potential, $V(x, y, z) = ?$

$$\Delta V = xa_x + ya_y + za_z \quad \dots (1)$$

We have,

$$\Delta V = \frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z \quad \dots (2)$$

From equations (1) and (2),

$$\frac{\partial V}{\partial x} = x \quad \dots (3)$$

$$\frac{\partial V}{\partial y} = y \quad \dots (4)$$

$$\frac{\partial V}{\partial z} = z \quad \dots (5)$$

Integrating equations (3), (4) and (5) we have,

$$V = \frac{x^2}{2} + k \quad \dots (6)$$

$$V = \frac{y^2}{2} + k \quad \dots (7)$$

$$V = \frac{z^2}{2} + k \quad \dots (8)$$

$$V(x, y, z) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} + k$$

Where, k = Constant.

The point (1, 1, 1) m is at 0 V.

Now,

$$V(x, y, z) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} + k$$

$$0 = \frac{1^2}{2} + \frac{1^2}{2} + \frac{1^2}{2} + k$$

$$0 = 1.5 + k$$

$$\therefore \quad k = -1.5$$

$\therefore$ The potential $V(x, y, z)$ is the point (1, 1, 1) m is at 0 V is,

$$V(x, y, z) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} - 1.5$$

1.8 Potential Gradient

Q36. Potential for a certain region is given by $V(x, y, z) = \dfrac{300}{x}$ volts, where x is in meters. Find the electric field at the point P:(x = 1 m).

Feb.-08, Set-1, Q1(c) M[4]

Nov.-07, Set-3, Q1(c) M[4]

Answer :

Given data,

Potential $V(x, y, z) = \dfrac{300}{x}$

Point, P = 1 m

Electric field, $\overline{E}$ = ?

The electric field intensity 'E' at any point is negative gradient of potential,

$$\overline{E} = -\nabla . V$$

∇V can be expanded in 'x' coordinate system as,

$$\nabla.V = \frac{\partial V}{\partial x}\,\bar{a}_x$$

$\therefore$ Electric field intensity,

$$\bar{E} = -\left[\frac{\partial V}{\partial x}\bar{a}_x\right] \quad \text{... (1)}$$

Solving the differential term,

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left[\frac{300}{x^2}\right]$$

$$\frac{\partial V}{\partial x} = \frac{-300}{x^2} \quad \text{... (2)}$$

Substituting equation (2) in equation (1),

$$\bar{E} = -\left[\frac{\partial V}{\partial x}\bar{a}_x\right]$$

$$= -\left[\frac{-300}{x^2}\right]\bar{a}_x$$

$\therefore$ $\bar{E}$ at point $P(x = 1 \text{ m}) \Rightarrow P(1)$ m.

$$\therefore \quad \bar{E}_{(1)} = -\left[\frac{-300}{(1)^2}\right]\bar{a}_x$$

$$\bar{E}_{(1)} = 300\ \bar{a}_x \text{ V/m}$$

$$\boxed{\bar{E} = 300\bar{a}_x \text{ V/m}}$$

Q37. Potential for a certain region is given by

$$\mathbf{V(x, y, z) = \frac{300}{x} + \sin 0.1y + \log_e xy \text{ volts,}}$$

where x and y are in meters. Find the electric field at the point P : (x = 1 m, y = 0.6 m, z = 0).

Nov.-06, Set-1, Q1(c) M[6]

Answer :

Given data,

Potential, $V(x, y, z) = \frac{300}{x} + \sin 0.1\,y + \log_e xy$ V

Point, $P = (1, 0.6, 0)$ m

Electric field, $\bar{E} = ?$

The electric field intensity 'E' at any point is negative gradient of potential,

$$\bar{E} = -\nabla.V$$

$\nabla.V$ can be expanded in xyz coordinate system as,

$$\nabla.V = \frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z$$

$\therefore$ Electric field intensity,

$$\bar{E} = -\left(\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right) \quad \text{... (1)}$$

Solving for differential terms,

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{300}{x} + \sin 0.1y + \log_e xy\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{300}{x^2} + \sin 0.1y + 2.303\log_{10} xy\right)$$

$$= \frac{-300}{x^2} + 0 + \frac{2.303}{xy}.y$$

$$= \frac{-300}{x^2} + \frac{2.303}{x}$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}\left(\frac{300}{x} + \sin 0.1y + \log_e xy\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{300}{x} + \sin 0.1y + 2.303\log_{10} xy\right)$$

$$= 0 + 0.1 \cos 0.1\,y + \frac{2.303}{xy}.x$$

$$= 0.1 \cos 0.1\,y + \frac{2.303}{y}$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}\left(\frac{300}{x} + \sin 0.1y + \log_e xy\right)$$

$$= 0$$

Substituting the above values in equation (1),we get,

$$\bar{E} = -\left(\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right)$$

$$= -\left[\left(\frac{-300}{x^2}+\frac{2.303}{x}\right)\bar{a}_x+\left(0.1\cos 0.1y+\frac{2.303}{y}\right)\bar{a}_y+0\,\bar{a}_z\right]$$

$$= -\left[\left(\frac{-300}{x^2}+\frac{2.303}{x}\right)\bar{a}_x+\left(0.1\cos 0.1y+\frac{2.303}{y}\right)\bar{a}_y\right]$$

$\therefore$ $\bar{E}$ at point $P(1, 0.6, 0)$ m is,

$$\bar{E}_{(1,0.6,0)} = -\left[\left(\frac{-300}{(1)^2}+\frac{2.303}{1}\right)\bar{a}_x+\left(0.1\cos 0.1(0.6)+\frac{2.303}{0.6}\right)\bar{a}_y\right]$$

$$= -[(-300 + 2.303)\,\bar{a}_x + (0.059 + 3.83)\,\bar{a}_y$$

$$= 297.69\,\bar{a}_x - 3.92\,\bar{a}_y \text{ V/m}$$

1.9 Gauss's Law

Q38. State and explain Gauss's law.

March-06, Set-3, Q2(a) M[8]
March-06, Set-4, Q2(a) M[8]
Nov.-05, Set-2, Q2(a) M[8]
Nov.-05, Set-3, Q2(a) M[8]

Answer :

Statement

It states that the net flux coming out of any closed surface is equal to the total charge enclosed by that surface.

Mathematically,

$$\boxed{\psi = Q}$$

Where,

ψ = Net flux coming out of the surface.

Q = Charge enclosed by the surface.

Explanation

Consider a charge 'Q' enclosed at the centre of a sphere of radius 'r' as shown in the figure. The charge 'Q' causes the 'Q' coulombs of flux and hence electric flux density exists on the surface of the sphere. Here, instead of sphere, any closed surface can be assumed. Consider a differential surface area $d\bar{s}$ on the sphere as shown in the figure.

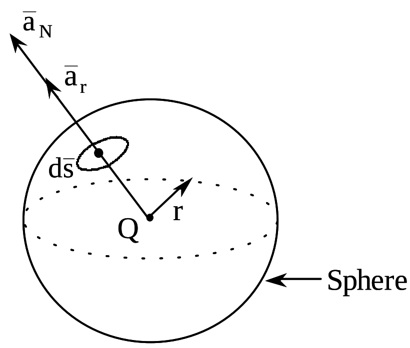


Figure: Charge 'Q' Enclosed by a Sphere

Let $\overline{a}_N$ be the unit vector passing through the differential surface area $d\overline{s}$ and whose direction is normal to the surface of the sphere.

$\therefore \quad \overline{a}_r \cdot \overline{a}_N = 1$

We know that, electric field density ($\overline{D}$) is given by,

$$\overline{D} = \in_0 \overline{E} \qquad \dots (1)$$

By the definition for a sphere, we have,

$$\overline{E} = \frac{Q}{4\pi\in_0 r^2}\overline{a}_r \qquad \dots (2)$$

Substituting equation (2) in equation (1),

$$\overline{D} = \in_0 \times \frac{Q}{4\pi\in_0 r^2}\overline{a}_r$$

$$\overline{D} = \frac{Q}{4\pi r^2}\overline{a}_r \qquad \dots (3)$$

Applying dot product with $d\overline{s}$ on both sides of the equation (3), we get,

$$\overline{D} \cdot d\overline{s} = \frac{Q}{4\pi r^2}.\overline{a}_r .ds\ \overline{a}_N$$

$$\overline{D} \cdot d\overline{s} = \frac{Q}{4\pi r^2} d\overline{s} \qquad \dots (4)$$

Applying surface integral on both sides, we get,

$$\oint_s \overline{D}.d\overline{s} = \oint_s \frac{Q}{4\pi r^2}\ d\overline{s}$$

$$\oint_s \overline{D}.d\overline{s} = \frac{Q}{4\pi r^2}\oint_s d\overline{s}$$

But, $\oint_s d\overline{s}$ = Surface area of the sphere = $4\pi r^2$

$$\oint_s \overline{D}.d\overline{s} = \frac{Q}{4\pi r^2} \times 4\pi r^2$$

$$\oint_s \overline{D}.d\overline{s} = Q \qquad \dots (5)$$

Where,

$\oint_s \overline{D}.d\overline{s}$ = Total flux passing through the surface = ψ

$$\boxed{\therefore\ \psi = Q}$$

Hence, it can be inferred that the total flux (ψ) passing through the surface is equal to the charge (Q) enclosed by that surface.

Q39. State and prove Gauss's law in integral form, considering static charges in free space.

Nov.-06, Set-4, Q1(a) M[6]

Answer :

For answer refer Unit-I, Q38.

1.10 Applications of Gauss's Law

Q40. Using Gauss law, find E at any point due to long infinite charge wire.

March-06, Set-3, Q2(b) M[8]

March-06, Set-4, Q2(b) M[8]

Nov.-05, Set-2, Q2(b) M[8]

Nov.-05, Set-3, Q2(b) M[8]

Answer :

Electric Field Due to Long Infinite Charge Wire

Consider a conducting wire, carrying a uniformly distributed charge 'Q' of linear charge density 'ρ_l' is placed along the Z-axis as shown in the figure. Consider a cylindrical Gaussian surface around the infinite line charge so that it is symmetrical about the infinite line charge as depicted in the figure. Let 'r' be the radius of the cylinder and 'l' be its length.

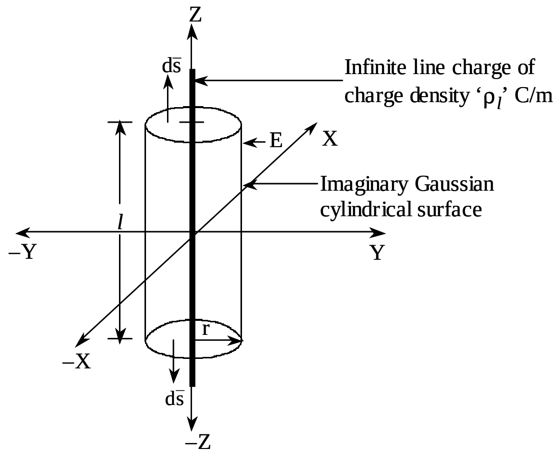


Figure: Imaginary Cylindrical Gaussian Surface Around the Line Charge

For answer refer Unit-I, Q30, Topic: Derivation.

$\therefore$ Above equation gives the field intensity at any point due to infinite length line charge.

Q41. A charge of Q is distributed uniformly throughout the volume of a sphere of radius R mts. Find electric field at any point using Gauss law.

Nov.-05, Set-4, Q2(a) M[10]

Answer :

Electric Field at any Point on a Uniformly Charged Sphere of Radius 'R' Using Gauss's Law

Consider a sphere of radius 'R' mts with a charge 'Q' uniformly distributed throughout the volume of the sphere as shown in figure (a). Let 'ρ_v' denotes the volume charge density of the sphere.

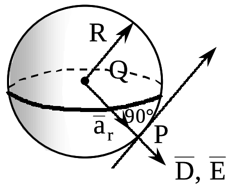


Figure (a): Sphere of radius 'R'

Electric field intensity $(\bar{E})$ at any point on the surface of the sphere is normal to the tangent drawn at that point. Let the medium is assumed to be isotropic in nature, therefore, the direction of electric flux density $(\bar{D})$ and electric field intensity $(\bar{E})$ is same.

The electric field intensity $(\bar{E})$ at any point can be found by considering the following two cases.

Case (i)

Construct an imaginary Gaussian surface of radius 'a' such that $a \le R$, as shown in figure (b).

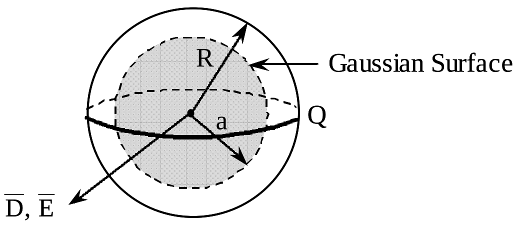


Figure (b): Imaginary Gaussian Surface of Radius 'a'

For $a \le R$, the volume of the sphere is the same. However, in the figure $a < R$ is considered.

According to the Gauss's law, total flux coming out of any closed surface is equal to the charge enclosed by that surface.

Mathematically,

$$\oint_s \bar{D}.d\bar{s} = Q \quad \ldots (1)$$

$$\Rightarrow \quad \in \oint_s \bar{E}.d\bar{s} = Q \qquad [\because \bar{D} = \in_0 \bar{E}]$$

Total charge enclosed by the sphere of radius 'r' is given by,

$$Q = \int_{vol} \rho_v .dv$$

$$\Rightarrow \quad Q = \rho_v \int_{vol} dv$$

$$\Rightarrow \quad Q = \rho_v \left[\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} r^2 \sin\theta dr d\theta d\phi \right]$$

[$\because$ Using spherical coordinates]

$$\Rightarrow \quad Q = \rho_v \left[\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left[\frac{r^3}{3} \right]_0^a \sin\theta d\theta d\phi \right]$$

$$\Rightarrow \quad Q = \rho_v \left[\int_{\phi=0}^{2\pi} d\phi \times \int_{\theta=0}^{\pi} \sin\theta .d\theta \left(\frac{a^3}{3} \right) \right]$$

$$\Rightarrow \quad Q = \rho_v \left[(2\pi - 0)(+2) \left(\frac{a^3}{3} \right) \right]$$

$$\Rightarrow \quad Q = \rho_v \frac{4\pi}{3} a^3 \quad \ldots (2)$$

Total flux coming out of the Gaussian sphere is,

$$\oint_s \bar{D}.d\bar{s} = \oint_s D_s \bar{a}_r .ds\bar{a}_r$$

$$\Rightarrow \quad \oint_s \bar{D}.d\bar{s} = D_s \left[\oint_s a^2 \sin\theta d\theta d\phi \right]$$

$$\Rightarrow \quad \oint_s \bar{D}.d\bar{s} = D_s a^2 \left[\int_{\theta=0}^{\pi} \left(\int_{\phi=0}^{2\pi} d\phi \right) \sin\theta .d\theta \right]$$

$$\Rightarrow \quad \oint_s \bar{D}.d\bar{s} = D_s a^2 [(2\pi - 0)\ (2)]$$

$$\Rightarrow \quad \oint_s \bar{D}.d\bar{s} = 4\pi a^2 D_s$$

$$\Rightarrow \quad \oint_s \bar{D}.d\bar{s} = 4\pi a^2 D_s \quad \ldots (3)$$

Substituting equations (2) and (3) in equation (1), we get,

$$4\pi a^2 D_s = \rho_v \frac{4\pi}{3} a^3$$

$$\Rightarrow \quad D_s = \frac{\rho_v \left(\frac{4\pi}{3} a^3 \right)}{4\pi a^2}$$

$$\Rightarrow \quad D_s = \rho_v \frac{a}{3}$$

$$\Rightarrow \quad \overline{D} = \rho_v \frac{a}{3}\overline{a}_r$$

[∵ Flux density is normal to the surface of the sphere]

$$\therefore \quad \overline{E} = \frac{\rho_v a}{3\epsilon_0}\overline{a}_r \text{ V/m for } 0 < r \le R \quad \ldots (4)$$

Case (ii)

Construct an imaginary Gaussian surface of radius 'a' such that $a > R$ as shown in the figure (c).

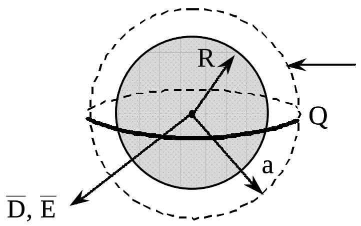


Figure (c): Imaginary Gaussain Sphere of Radius a > R

Here, the charge enclosed by the Gaussian surface of radius 'a' is the total charge enclosed by the sphere of radius 'R'.

$$\text{i.e.,} \quad Q = \int_{vol} \rho_v . dv$$

$$\Rightarrow \quad Q = \rho_v \int_{vol} dv$$

$$\Rightarrow \quad Q = \rho_v \left[\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{R} r^2 \sin\theta . d\theta d\phi dr\right]$$

$$\Rightarrow \quad Q = \rho_v \left[(2\pi - 0)[-\cos\pi + \cos 0]\left(\frac{R^3}{3}\right)\right]$$

$$\Rightarrow \quad Q = \rho_v \left[(2\pi)(2)\left(\frac{R^3}{3}\right)\right]$$

$$\Rightarrow \quad Q = \frac{4}{3}\pi R^3 \times \rho_v$$

Total flux coming out of the Gaussian surface of radius 'a' is given by,

$$\oint_s \overline{D}.d\overline{s} = \oint_s \overline{D}_s.\overline{a}_r.d\overline{s}.\overline{a}_r$$

$$= D_s \oint_s d\overline{s}$$

$$= D_s \left[\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} a^2 \sin\theta . d\theta . d\phi\right]$$

$$= D_s a^2 \left[\left(\int_{\phi=0}^{2\pi} . d\phi\right)\left(\int_{\phi=0}^{\pi} \sin\theta . d\theta\right)\right]$$

$$= D_s a^2[(2\pi - 0)(-\cos\pi + \cos 0)]$$

$$= D_s a^2 [2\pi \times 2]$$

$$\therefore \quad \oint_s \overline{D}.d\overline{s} = 4\pi a^2 D_s \quad \ldots (5)$$

$$4\pi a^2 D_s = \frac{4}{3}\pi R^3 \rho_v$$

$$\Rightarrow \quad D_s = \frac{R^3 \rho_v}{3a^2}$$

$$\therefore \quad \overline{D} = \frac{R^3 \rho_v}{3a^2}$$

$$\therefore \quad \overline{E} = \frac{R^3 \rho_v}{3\epsilon_0 a^2}\overline{a}_r \text{ V/m}$$

∴ The results can be summarized as,

$$\overline{E} = \begin{cases} \dfrac{a}{3\epsilon_0}\rho_v \overline{a}_r \text{ V/m} & ; 0 < a \le R \\ \dfrac{R^3}{3\epsilon_0 a^2}\rho_v \overline{a}_r \text{ V/m} & ; a \ge R \end{cases}$$

Q42. Find electric field at any point due to infinite charge surface using Gauss law.

Nov.-05, Set-4, Q2(b) M[6]

OR

Show that the electric field intensity due to an infinite sheet of charge is independent of the distance of the point from sheet.

Nov.-04, Set-4, Q1(a)

Answer :

For answer refer Unit-I, Q32.

Q43. Find the electric field at a point outside a sphere of radius R m in free space which carries a uniform charge density ρ C/m³. Use Gauss's law.

Feb.-08, Set-3, Q1(b) M[6]

Answer :

For answer refer Unit-I, Q41, Topic: Case (ii).

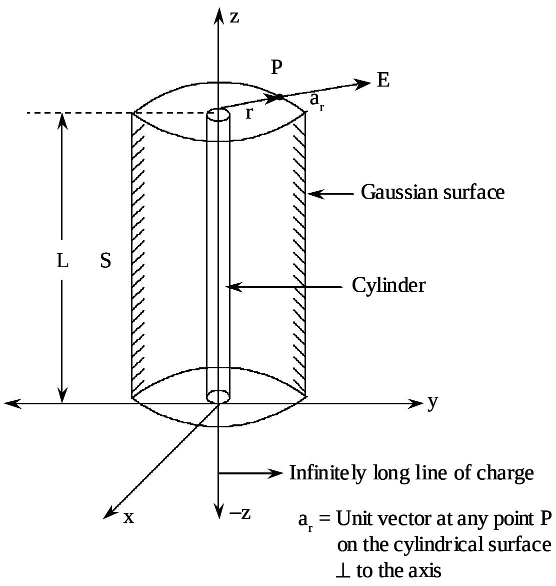
Q44. Using Gauss's law, show that the electric field due to an infinite straight line of uniform charge density λ C/m along the z-axis in free space is $(\lambda/2\pi\epsilon_0 r)$ a_r N/C.

Feb.-08, Set-4, Q1(b) M[6]

Answer :

Electric Field Due to Infinite Line

Consider a conductor of infinite length. Let 'λ' C/m be the line charge density. Assuming that the conductor is lying along the z-axis as shown in figure.



Figure

The conductor carries a uniformly distributed charge, the charge density being 'λ' C/m. Assume a Gaussian surface around an infinite line charge, which is imagined co-axial cylinder. Let '*r*' be the radius of the cylinder and '*L*' be the length of the cylinder.

The directions of electric flux density '*D*' and electric field intensity '*E*' are radial everywhere.

∴ Total charge, Q = Length × Charge density

$$= L \times \lambda$$

According to Gauss's law, total flux coming out from a closed surface is equal to the charge enclosed by the surface area.

i.e., $$Q = \oint_S \bar{D}.d\bar{s}$$

Here, the imaginary cylinder comprises of three areas, i.e., top, bottom and curved surface area. So, the total flux is the sum of the fluxes coming out from these three areas.

$$Q = \oint_{\text{Curved}} \bar{D}.d\bar{s} + \oint_{\text{Top}} \bar{D}.d\bar{s} + \oint_{\text{Bottom}} \bar{D}.d\bar{s}$$

$$= D \int_{\text{Curved}} ds + \int_{\text{Top}} ds + \int_{\text{Bottom}} ds$$

∵ Components for the top and bottom surfaces are zero.

$$\therefore \quad Q = D \int_{\text{Curved}} ds = D \int_0^L 2\pi r dL$$

$$Q = D(2\pi rL) \quad \Rightarrow \quad D = \frac{Q}{2\pi rL}$$

As, $Q = L \times \lambda$

$$D = \frac{L \times \lambda}{2\pi rL} = \frac{\lambda}{2\pi r} \qquad \text{... (1)}$$

We know that,

$$D = \epsilon_0 E$$

$$\Rightarrow \quad E = \frac{D}{\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 r}$$

At a distance '*r*' field intensity is,

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r} a_r \text{ N/C}}$$

1.11 Maxwell's First Law div(D) = ρ_v

Q45. State and prove Maxwell's first law for electrostatic fields in its point and integral forms.

Answer :

Maxwell's First Law

Statement

Maxwell's first law states that the total electric flux passing through any closed surface is equal to the total volume charge enclosed by that surface.

In point form it is expressed as,

$$\nabla.\bar{D} = \rho_v$$

Where,

$\bar{D}$ = Electric flux density

ρ_v = Volume charge density.

In integral form it is expressed as,

$$\oint_s \bar{D}.d\bar{s} = \int_v \rho_v \, dv$$

Proof

Maxwell's first law is derived from Gauss's law.

According to Gauss's law, the electric flux coming out of any closed surface is equal to the total charge enclosed by that surface.

Mathematically,

$$\psi = \oint_s \overline{D}.d\overline{s} = Q \quad \text{... (1)}$$

Where,

ψ = Electric flux

$\overline{D}$ = Electric flux density

Q = Total charge enclosed.

By definition we have, total charge enclosed by a volume,

$$Q = \int_v \rho_v \, dv \quad \text{... (2)}$$

From equations (1) and (2) we have,

$$\oint_s \overline{D}.d\overline{s} = \int_v \rho_v \, dv \quad \text{... (3)}$$

Applying divergence theorem to L.H.S term, we get,

$$\oint_s \overline{D}.d\overline{s} = \int_v (\nabla.\overline{D}) \, dv$$

$$\Rightarrow \int_v (\nabla.\overline{D}) \, dv = \int_v \rho_v dv$$

$$\therefore \quad \nabla.\overline{D} = \rho_v \quad \text{... (4)}$$

$\therefore$ Equations (4) and (3) are the Maxwell's first equation in point and integral forms respectively.

Q46. Derive $\nabla .D = \rho_v$ from fundamentals.

March-06, Set-3, Q1(a) M[8]

Answer :

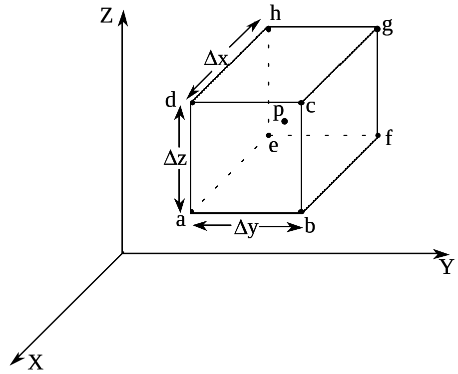
Consider a point charge of 'Q'C at point 'p' in XYZ plane. In order to find the flux coming out of this point charge using Gauss's law, there is no symmetry available either in rectangular or cylindrical or spherical coordinate system. But still, we can consider any closed surface with point 'p' as its centre and with its volume striking to zero.

So, consider a cube with sides Δx, Δy and Δz.

$\therefore$ Volume of the cube, $\Delta v = \Delta x \, \Delta y \, \Delta z$

$\therefore$ The flux leaving the cubical volume is,

$$\psi = \oiint \overline{D} d\,\overline{s}$$



Figure

The integral of electric flux density over the entire closed region can be obtained by adding the flux leaving from all the six sides of the cube.

Let, the flux leaving from the front (*abcd*), back (*efgh*), left (*aehd*), right (*bfgc*), top (*dcgh*) and bottom (*abfe*) sides of the cube be ψ_1, ψ_2, ψ_3, ψ_4, ψ_5 and ψ_6 respectively.

$$\therefore \quad \psi = \oiint \overline{D} d\,\overline{s}$$

$$= \psi_1 + \psi_2 + \psi_3 + \psi_4 + \psi_5 + \psi_6 \quad \text{... (1)}$$

Flux leaving the front (*abcd*) side,

$$\psi_1 = \int \overline{D}_1.d\,\overline{s}_1$$

$$= \overline{D}_1.\int d\,\overline{s}_1$$

$$= \overline{D}_1.\Delta y \, \Delta z \; \overline{a}_x$$

$$= \Delta y \, \Delta z \; \overline{D}_1 . \; \overline{a}_x$$

$$= \Delta y \, \Delta z \, D_{1x}$$

As, D_{1x} is the normal component of the flux density at the front side ($\overline{D}_1$), it can be represented by an infinite series, expressed in terms of D_x, using Taylor's theorem as,

$$D_{1x} = D_x + \frac{\partial D_x}{\partial x}\frac{d}{1!} + \frac{\partial^2 D_x}{\partial x^2}\frac{(d)^2}{2!} + \ldots.$$

Where, d = Distance between point p and front side (centre).

$$d = \frac{\Delta x}{2}$$

$$\therefore \quad D_{1x} = D_x + \left(\frac{\partial D_x}{\partial x} \times \frac{\Delta x/2}{1!}\right) + \left(\frac{\partial^2 D_x}{\partial x^2} \times \frac{(\Delta x/2)^2}{2!}\right) + \ldots\ldots$$

Neglecting all the second and higher order terms, as $\frac{\Delta x}{2}$ is too small.

$$\therefore \quad D_{1x} \cong D_x + \frac{\partial D_x}{\partial x}\frac{\Delta x}{2} \qquad \ldots (2)$$

$$\therefore \quad \psi_1 = \Delta y\, \Delta z \left(D_x + \frac{\partial D_x}{\partial x}\frac{\Delta x}{2}\right)$$

$$= D_x\, \Delta y\, \Delta z + \frac{\partial D_x}{\partial x}\frac{\Delta x \Delta y \Delta z}{2}$$

$$= D_x\, \Delta y\, \Delta z + \frac{\partial D_x}{\partial x}\frac{\Delta v}{2} \qquad \ldots (3)$$

Flux leaving the back (*efgh*) side,

$$\psi_2 = \int \overline{D}_2 . d\overline{s}_2$$

$$= \overline{D}_2 . \int d\overline{s}_2$$

$$= \overline{D}_2 . \Delta y\, \Delta z\, (-\, \overline{a}_x)$$

$$= -\, \Delta y\, \Delta z\, \overline{D}_2 . \overline{a}_x$$

$$= -\, \Delta y\, \Delta z\, D_{2x}$$

As, the normal component of the flux density at the back side (D_{2x}) is accelerating towards negative x-axis, the term $\frac{\partial D_x}{\partial x}$ of the Taylor's series will be negative.

$$\therefore \quad D_{2x} \cong D_x + \frac{\partial D_x}{\partial x}\frac{\Delta x}{2}$$

$$\therefore \quad \psi_2 = -\Delta y\, \Delta z \left(D_x + \frac{\partial D_x}{\partial x}\frac{\Delta x}{2}\right)$$

$$= -\, D_x\, \Delta y\, \Delta z + \frac{\partial D_x}{\partial x}\frac{\Delta x \Delta y \Delta z}{2}$$

$$= -\, D_x\, \Delta y\, \Delta z + \frac{\partial D_x}{\partial x}\frac{\Delta v}{2}$$

$$\ldots (4)$$

The sum of equations (3) and (4) i.e., the sum of flux leaving the front and back sides of the cube will give the net flux leaving the cube along x-direction.

$$\therefore \quad \psi_x = \psi_1 + \psi_2$$

$$= D_x \Delta y \Delta z + \frac{\partial D_x}{\partial x}\frac{\Delta v}{2} - D_x \Delta y \Delta z + \frac{\partial D_x}{\partial x}\frac{\Delta v}{2}$$

$$= \frac{\partial D_x}{\partial x}\Delta v \quad \dots (5)$$

Similarly, the net flux leaving the cube along y-direction,

$$\psi_y = \psi_3 + \psi_4 = \frac{\partial D_y}{\partial y}\Delta v \quad \dots (6)$$

The net flux leaving the cube along z-direction,

$$\psi_z = \psi_5 + \psi_6 = \frac{\partial D_z}{\partial z}\Delta v \quad \dots (7)$$

Substituting equations (5), (6) and (7) in equation (1), we get,

$$\psi = \frac{\partial D_x}{\partial x}\Delta v + \frac{\partial D_y}{\partial y}\Delta v + \frac{\partial D_z}{\partial z}\Delta v$$

$$\therefore \quad \psi = \Delta v\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \quad \dots (8)$$

But, from Gauss's law, we have the total flux coming out of this cubical volume equals the total charge enclosed by it.

i.e., $\psi = Q$ ... (9)

From equations (8) and (9) we have,

$$\Delta v\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = Q$$

$$\Rightarrow \quad \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{Q}{\Delta v}$$

$$\Rightarrow \quad \nabla.\overline{D} = \frac{Q}{\Delta v}$$

When $\Delta v \to 0$, $\frac{Q}{\Delta v}$ will be the volume charge density 'ρ_v' (associated with the point charge at point 'p').

$$\therefore \quad \underset{Lt \to 0}{\Delta v}\frac{Q}{\Delta v} = \rho_v$$

$$\Rightarrow \quad \nabla.\overline{D} = \rho_v$$

The above equation is called as the Maxwell's first law or the point form of Gauss's law.

Q47. Define the term: "Potential difference V(A) – V(B) between points A and B in a static electric field". Give an energy interpretation to potential difference.

Feb.-08, Set-1, Q1(a) M[6]

Answer :

For answer refer Unit-I, Q26, Topic: Potential Difference.

Energy Interpretation of Potential Difference

The potential difference between two points is given by,

$$V_{AB} = \int_B^A \overline{E}.d\overline{L}$$

$$\overline{E} = \frac{Q}{4\pi\epsilon_0 R^2}$$

$$\therefore \quad V_{AB} = \int_{R_B}^{R_A} \frac{Q}{4\pi\epsilon_0 R^2} d\overline{R}$$

$$= \frac{Q}{4\pi\epsilon_0}\int_{R_B}^{R_A}\frac{1}{R^2}d\overline{R}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{R}\right]_{R_B}^{R_A}$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R_B} - \frac{1}{R_A}\right] \quad \dots (1)$$

'Energy' is the total power consumed or delivered over a time interval 't' i.e., electrical energy developed as work or dissipated as heat over a time interval 't' is given as,

$$W = \int_0^t VI\,dt$$

$$\therefore \quad W = \int_0^t \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R_B} - \frac{1}{R_A}\right] I\,dt$$

The unit of energy is Joule or Watt sec.