SPONTANEOUS BREAKDOWN OF STRONG INTERACTION SYMMETRY AND THE ABSENCE OF MASSLESS PARTICLES

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The occurrence of massless particles in the presence of spontaneous symmetry breakdown is discussed. By summing all Feynman diagrams, one obtains for the difference of the mass operators $M_a(p) - M_b(p)$ of particles a and b belonging to a supermultiplet an equation which is identical to the Bethe-Salpeter equation for the wave function of a scalar bound state of vanishing mass (a "zeron") in the annihilation channel $\bar{a}b$ of the corresponding particles. It is shown that if symmetry is spontaneously violated in a Yang–Mills type theory involving vector mesons, the zerons interact only with virtual particles and therefore unobservable. On the other hand, the vector mesons acquire a mass in spite of the generalized gauge invariance. It is shown in Appendices A and B that the asymmetrical solution corresponds to a minimal energy of the vacuum and that C-invariance of the solution implies strangeness conservation for it.

1. INTRODUCTION

SPONTANEOUS symmetry breakdown related to an instability of the system under consideration with respect to an infinitesimally weak asymmetric perturbation is often encountered in quantum statistical mechanics (ferromagnetism, superconductivity, etc.). A large number of such examples has been analyzed in Bogolyubov's review.[1]

In the theory of elementary particles the possibility of spontaneous asymmetry has been discussed by Nambu and Jona–Lasinio[2] and by Vaks and Larkin,[3] on the example of violated γ_5-invariance. In their calculations these authors ran into the existence of massless particles (zerons). Later, Goldstone has proved the theorem[4, 5] which states that bound states of zero mass must appear in systems with unstable symmetries. An example are the spin waves which occur in a ferromagnetic body and the acoustic oscillations in a boson gas. On the other hand, there are no acoustic oscillations in a superconductor. Lange[6] has shown that the cause for the inapplicability of the Goldstone theorem in this case are the long-range Coulomb forces.

In elementary particle theory, forces analogous to the Coulomb forces appear as a result of the exchange of massless Yang–Mills vector mesons[7] which guarantee SU(3) symmetry and generalized gauge invariance (cf. e.g., the paper of Glashow and Gell-Mann[8]). It is to be expected that no zerons would appear with spontaneous symmetry breakdown in such a theory, and that the vector mesons acquire a physical mass in analogy with the screening of Coulomb forces in a superconductor.

We show that the vector mesons do indeed acquire a mass and that the zerons do not interact with real particles. Such a "phantom" zeron manifests itself as an off-mass-shell pole in the scattering amplitudes. The scattering amplitudes for real particles do not have such poles and therefore the zerons are unobservable. We note that in a Yang–Mills theory with stable symmetry the vector mesons would not have a physical mass; the instability of the symmetry removes this difficulty, which has been repeatedly discussed in the literature.[9]

Our paper is organized in the following manner: In Sec. 2 the equations of Nambu[2] and Vaks and Larkin,[3] which were introduced for weak point interactions, are extended to the case of an arbitrary interaction. The Goldstone theorem[4] is confirmed for this case: a condition for the solvability of the equations so derived is the existence of a bound state of zero mass (zeron). In Sec. 3 we prove the unobservability of zerons in a theory with vector mesons. In Appendix A the necessity of choosing (from energy considerations) the asymmetric solution, whenever it exists, is justified. Appendix B contains a proof of the connection between the conservation of strangeness and C-invariance in the presence of spontaneous symmetry breaking.

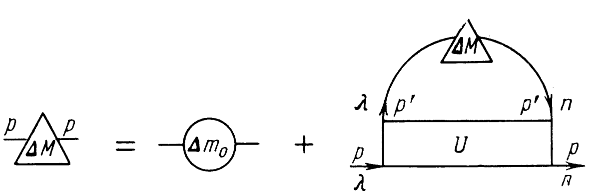
2. INSTABILITY OF SYMMETRY IN QUANTUM FIELD THEORY

We explain the meaning of instability of symmetry in quantum field theory without assuming that the interaction is weak. We discuss this problem on the example of a system and n- and λ-quarks with identical initial masses and interactions invariant under the SU(2) group, involving the n and λ axes.

Instability of the symmetry means that if one adds to the Hamiltonian an infinitesimally weak asymmetric perturbation, there appears in the system a finite symmetry-breaking effect. We take as such a perturbation a splitting between the bare masses of the n and the λ, to be denoted by Δm_0. The particles n and λ will now have different mass operators $M_n(p)$ and $M_\lambda(p)$. We look for a condition which guarantees that if the initial asymmetry is removed ($\Delta m_0 \to 0$) the physical asymmetry $(M_n(p) - M_\lambda(p))$ persists., For finite Δm_0 there exists a simple equation for the difference between $M_n(p)$ and $M_\lambda(p)$:

$$\Delta M(p) \equiv M_n(p) - M_\lambda(p) = \Delta m_0 + \int \frac{d^4p'}{(2\pi)^4} U(p,p',0) G_n(p') \Delta M(p') G_\lambda(p'), \qquad (2.1)$$

which corresponds to the Feynmann diagram



(2.1′)

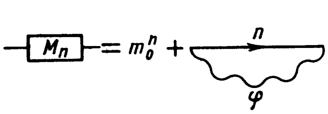
Here G_n and G_λ are the exact Green's functions of n and λ:

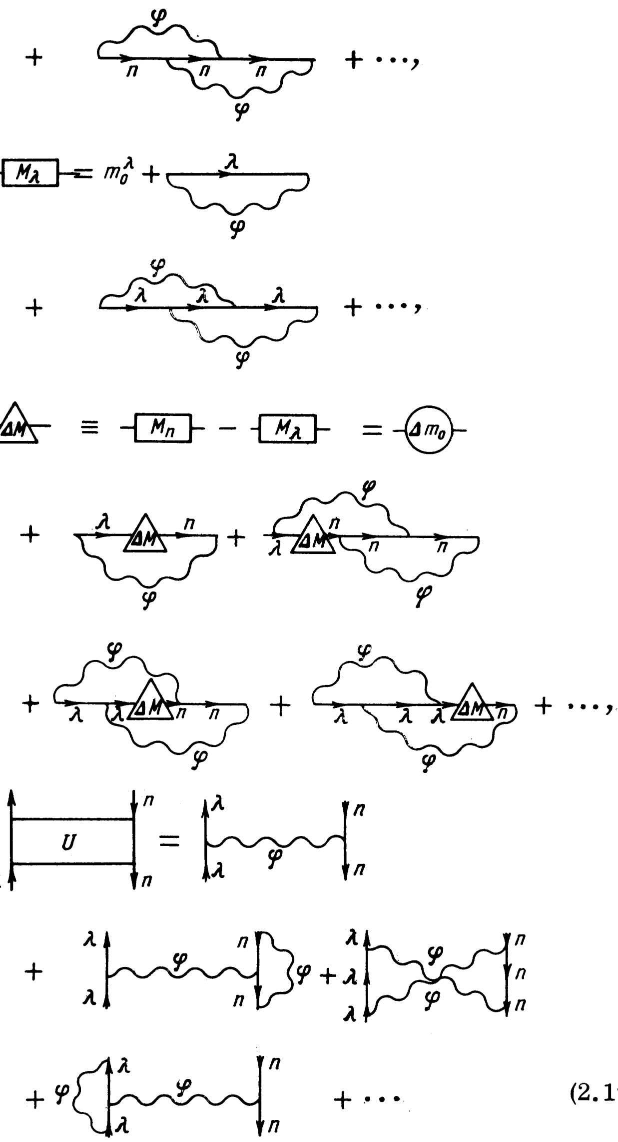
$$G_n(p) = [p_\mu\gamma^\mu - M_n(p)]^{-1},$$

$$G_\lambda(p) = [p_\mu\gamma^\mu - M_\lambda(p)]^{-1};$$

U(p, p′, q) is the set of all $n\bar{\lambda}$-scattering graphs containing no parts connected by n or $\bar{\lambda}$ lines only.

The equation (2.1) has been derived by simple diagram subtraction. A diagrammatic derivation of this equation for an interaction with an "isoscalar" φ meson, $(\bar{n}n + \bar{\lambda}\lambda)\varphi$, goes as follows:





(2.1″)

Here the straight lines denote the exact Green's functions of the n and the λ, and the wavy lines are the exact Green's functions of the φ meson. Use has been made of the identity

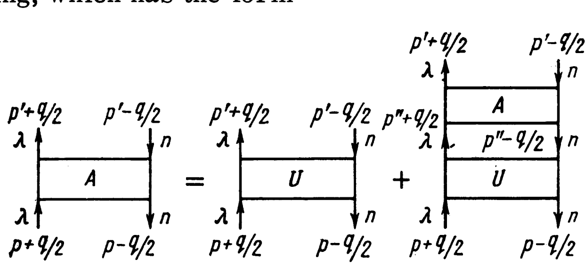
$$A_1A_2 \ldots A_n - B_1B_2 \ldots B_n = (A_1 - B_1)B_2 \ldots B_n + A_1(A_2 - B_2)B_3 \ldots B_n + A_1A_2 \ldots A_{n-1}(A_n - B_n).$$

In (2.1) we go to the limit $\Delta m_0 \to 0$. Then the equation for $\Delta M(p) = M_n(p) - M_\lambda(p)$ has the form

$$\Delta M(p) = \int \frac{d^4p'}{(2\pi)^4} U(p,p',0) G_n(p') \Delta M(p') G_\lambda(p'). \qquad (2.2)$$

(This equation is a generalization to arbitrary interactions of the model equations given in [2,3].)

The homogeneous equation (2.2) can have a nonvanishing solution corresponding to a spontaneous symmetry violation if the eigenvalue of the operator UG_nG_λ equals one.[1)] We show that the physical meaning of this condition for the instability of the symmetry is the existence of a scalar bound state of vanishing mass in the $\bar{\lambda}n$ channel. To this end we compare (2.2) with the Bethe-Salpeter equation for the amplitude A(p, p′, q) for $\bar{\lambda}n$ scattering, which has the form



(2.3)

or the analytic expression

$$A(p,p',q)=U(p,p',q)$$
$$+\int\frac{d^4p''}{(2\pi)^4}U(p,p'',q)G_n\left(p''-\frac{q}{2}\right)$$
$$\times A(p'',p',q)G_\lambda\left(p''+\frac{q}{2}\right). \qquad (2.3')$$

Letting $q_\alpha \to 0$ in (2.3) we see that the condition for the existence of a scalar pole at $q_\alpha = 0$ in the amplitude A(p, p′, q) and the condition that Eq. (2.2) for $\Delta M(p)$ admit a solution, are the same.

Thus, we can draw the conclusion that for an interaction in the $n\bar{\lambda}$ system sufficiently strong to form a zero mass bound state, Eq. (2.2) for the difference between the mass operators of the n and the λ admits a nonvanishing solution. This solution corresponds to spontaneous breakdown of the SU(2) symmetry and can be obtained by introducing an infinitesimal symmetry violation into the initial Hamiltonian.

Since, of course, Eq. (2.2) also admits zero as a solution, corresponding to non-violated symmetry, there arises the question as to how to choose the solution. This problem is discussed in Appendix A.

If one does not pose the question as to how a mass difference can appear in a system with a symmetric Hamiltonian, the existence of the zeron in itself can be established in a simpler manner, for instance by making use of the generalized Ward identity for the vertices involving conserved currents. In our case, for the vertex $J_\alpha(p, q)$ of the transition current describing the transition $\lambda \to n$, this identity becomes

$$q_\alpha J_\alpha(p,q) = G_\lambda^{-1}(p+q/2) - G_n^{-1}(p-q/2). \qquad (2.4)$$

Let $q_\alpha \to 0$ in (2.4)

$$\lim_{q_\alpha\to 0}[q_\alpha J_\alpha(p,q)] = M_n(p) - M_\lambda(p). \qquad (2.5)$$

The left-hand side in (2.5) does not vanish if $M_\lambda(p) \neq M_n(p)$, so that $J_\alpha(p, q)$ exhibits a scalar pole at $q_\alpha = 0$. This pole corresponds to the zeron. However we have seen this pole both in J_α and in A (cf. (2.3)) at equal momenta of the n and λ, i.e., off the mass shell, since $m_n \neq m_\lambda$. In the following section we show that in theories involving vector mesons this pole does not appear on the mass shell, and thus the zerons are fictitious.

3. THE ACQUISITION OF A MASS BY THE VECTOR MESONS AND THE DISAPPEARANCE OF THE ZERONS

In this section it will be shown that in a Yang–Mills type theory with generalized gauge invariance (cf. [7,8]) the zerons which appear due to spontaneous symmetry breakdown are unobservable, and that the vector mesons acquire a physical mass, even though they have no initial mass. We shall discuss these problems using the simple example of a system consisting of a doublet of quarks n and λ, interacting with a triplet of vector mesons ρ^0_U, K^{*0}, $\bar{K}^{*0}$. The system is assumed invariant under the group SU(2) of U-spin,[10] the mesons have a vanishing bare mass and interact with the vector current of the U-spin of all particles.

As explained in Sec. 2, spontaneous symmetry breakdown in the system produces a zeron in the $\bar{\lambda}n$-channel (the Goldstone theorem). Leaving aside for the moment the problem of the observability of zerons, we show that the zeron pole in the irreducible self-energy part of the K* meson leads to a nonvanishing physical mass for the K* meson.

Denoting the amplitude for the transition of a K* meson into a zeron at momentum q by $a(q^2)q_\alpha$, the contribution of the zeron to the irreducible self-energy part of the K* meson, $\Pi_{\alpha\beta}(q)$, has the form

$$\Pi_{\alpha\beta}(q) \xrightarrow[q^2\to 0]{} -a^2(q^2)q_\alpha q_\beta / q^2. \qquad (3.1)$$

[1)]One should not think that Eq. (2.2) has a solution only for a definite fixed value of the original parameters (mass, coupling constants, etc.). This is not so since (2.2) is nonlinear: UG_nG_λ depends functionally on $M_n(p)$ and $M_\lambda(p)$. Model examples and the analogy with the theory of superconductivity indicate that there is a solution if the original coupling constant corresponds to sufficiently strong attraction of the $\bar{\lambda}$ and n. As the attraction is increased, the mass difference persists and as a rule increases.

The pole (3.1) in $\Pi_{\alpha\beta}(q)$ is due to the K* meson–zeron transition (amplitude $a(q^2)q_\alpha$) followed by the propagation of the zeron (the Green's function $D(q^2) \to 1/q^2$) and the inverse zeron–K* meson transition (the factor $a(q^2)q_\alpha$).[2)] In order to understand the influence exerted by the zeron pole (3.1) on the mass of the K* meson we recall that the mass m_K is defined by the equation

$$m_K{}^2 = \Pi(m_K{}^2). \tag{3.2}$$

Here the mass operator $\Pi(q^2)$ is related to $\Pi_{\alpha\beta}(q)$ as follows

$$\Pi_{\alpha\beta}(q) = \Pi(q^2)(\delta_{\alpha\beta} - q_\alpha q_\beta q^{-2}). \tag{3.3}$$

(The transversality of $\Pi_{\alpha\beta}(q)$ is a consequence of the U-spin vector current conservation.)

Comparing (3.3) and (3.1) at $q^2 \to 0$ we find

$$\Pi(0) = a^2(0) \equiv a^2 \neq 0. \tag{3.4}$$

For $m_K = 0$ we would obtain from (3.2) $\Pi(0) = m_K^2 = 0$, thus (3.4) implies $m_K \neq 0$.

We stress that $a \neq 0$ due to the spontaneous symmetry. Indeed, the Ward identity for the vertex $\Gamma_{\alpha\beta}(p, q)$ describing the K* emission in the $\bar\lambda$-n quark annihilation (cf. Eqs. (2.4) and (2.5)[3)]) requires the existence of a pole in that vertex function, if $M_n(p) \neq M_\lambda(p)$. This pole can occur in Γ_α only if the zeron is capable of a transition into a K* meson, i.e., if $a \neq 0$. In a Yang–Mills theory with stable symmetry there is no zeron, hence

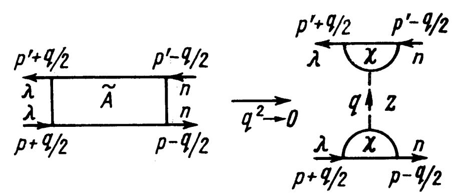
$$a^2 = \Pi(0) = m_K{}^2 = 0.$$

Thus one can draw the conclusion that the mass of a vector meson in a gauge invariant theory can appear if and only if the symmetry is spontaneously broken.

We go over to the proof of the assertion that the zeron does not participate in real processes and is therefore no obstacle for the theory of spontaneous symmetry breakdown in systems with generalized gauge invariance.

For the proof we consider the amplitude A for $\bar\lambda$n scattering and show that it does not contain zeron singularities on the mass shell (such singularities must exist off the mass shell, as seen from Eq. (2.3)). The absence of such singularities might appear strange, since they exist both in Γ_α and $\Pi_{\alpha\beta}$. However these quantities do not contain, by definition, diagrams exhibiting a K*-meson pole, whereas A contains such diagrams. If one removes these diagrams from A, the remaining amplitude $\tilde{A}$, which does not contain a K*-meson pole in the $\bar\lambda$n channel, will exhibit a zeron pole in the same channel. We show that the interference between the zeron pole in $\tilde{A}$ and the zeron of the other diagrams (which also exhibit K*-meson poles; the contribution of these diagrams will be denoted by $\tilde{\tilde{A}}$) cancels the resulting pole in $A = \tilde{A} + \tilde{\tilde{A}}$.

For $q^2 \to 0$ the amplitude $\tilde{A}$ has the form



(3.5)

or

$$\tilde{A}(p, p', q) \underset{q^2\to 0}{\longrightarrow} \frac{\chi(p, q)\chi(p', q)}{q^2}. \tag{3.5'}$$

(The amplitudes A, $\tilde{A}$, and $\tilde{\tilde{A}}$ depend on the three momenta p, p′, and q, cf. (3.5).) Here $\chi(p, q)$ denotes the sum of the diagrams which take the $\bar\lambda$n system into a zeron and do not exhibit a K*-pole in the $\bar\lambda$n-channel, since $\tilde{A}$ does not contain such a pole.

We express the residue in the zeron pole of the amplitude $\tilde{\tilde{A}}$ in terms of the same amplitude. To this end we rewrite the amplitude $\tilde{\tilde{A}}$ (before taking the limit $q^2 \to 0$) in the form

$$\tilde{\tilde{A}}(p, p', q) = \Gamma_\alpha(p, q)D_{\alpha\beta}(q)\Gamma_\beta(p', q). \tag{3.6}$$

Here $D_{\alpha\beta}(q) = \delta_{\alpha\beta}[\Pi(q^2) - q^2]^{-1}$ is the Green's function of the K* meson, $\Gamma_\alpha(p, q)$ is, as before, the sum of the $\bar\lambda$n–k* transition diagrams which do not exhibit a K*-meson pole.

If the $\bar\lambda$ and n are situated on the mass shell, the Ward identity (cf. footnote [3)]) implies that $\Gamma_\alpha(p, q)$ is transverse for all q^2 ($q_\alpha\Gamma_\alpha = 0$), hence on the mass shell A does not depend on the longitudinal part of $D_{\alpha\beta}(q)$, as was to be expected

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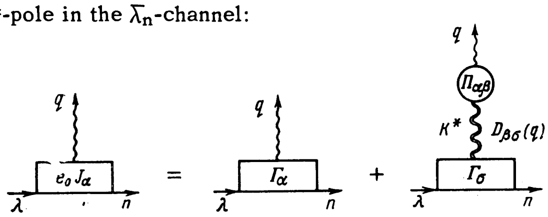
[2)]The minus sign in (3.1) is due to the fact that all amplitudes enter with an additional factor i in the Feynman amplitude.

[3)]A Ward identity of the form (2.4) can here be written for the current J_α differing from Γ_α by diagrams involving a K*-pole in the $\bar\lambda_n$-channel:



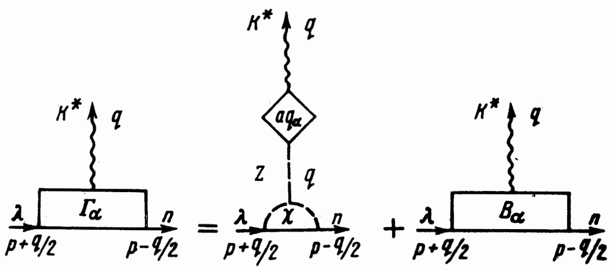
But owing to the transversality of $\Pi_{\alpha\beta}(q)$, (3.3), these diagrams do not contribute to the divergence, so that

$$e_0{}^{-1}q_\alpha\Gamma_\alpha(p, q) = q_\alpha J_\alpha(p, q) = G_\lambda{}^{-1}(p + q/2) - G_n{}^{-1}(p - q/2).$$

(here e_0 is the bare $n\lambda K^*$-coupling constant.

from gauge invariance. This has permitted us to work in the Feynman gauge for $D_{\alpha\beta}(q)$.

We now have to separate the pole in $\tilde{\tilde{A}}$ at $q^2 \to 0$. A zeron singularity in $\tilde{\tilde{A}}$ will appear, due to the zeron poles in the vertices Γ_α (we recall that $m_K \neq 0$, and therefore $D_{\alpha\beta}(q)$ does not exhibit a singularity at $q^2 \to 0$). The zeron poles in Γ_α appear as a result of the transition of the $\bar{\lambda}n$ into a zeron (described by the amplitude χ), the propagation of the zeron ($D(q^2) \to q^{-2}$), and its transition into a K* meson with the amplitude aq_α. Therefore Γ_α can be represented in the form

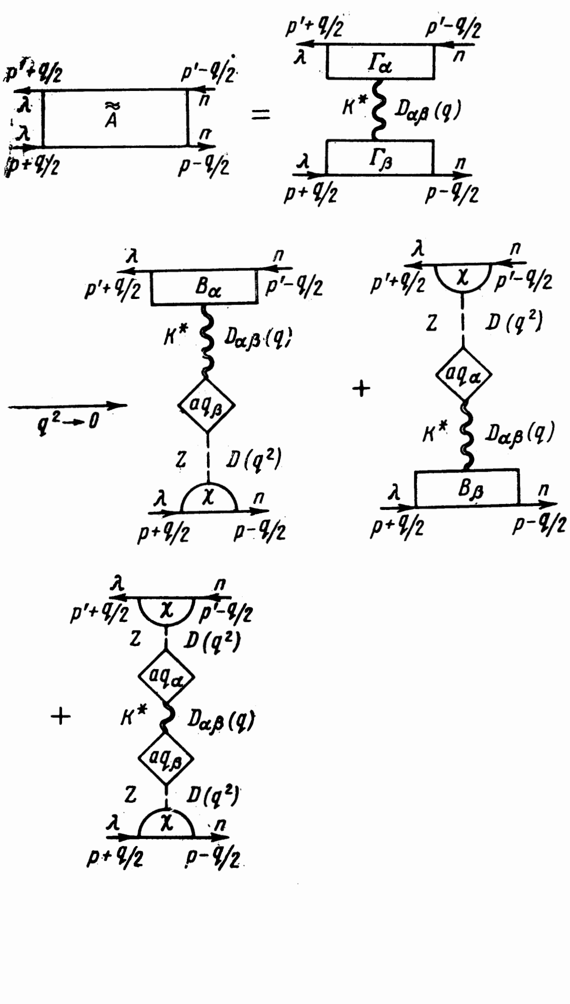


(3.7)

or

$$\Gamma_\alpha(p, q) = \chi(p, q) q^{-2} a q_\alpha + B_\alpha(p, q), \qquad (3.7')$$

where $B_\alpha(p, q)$ has no pole for $q^2 \to 0$. Substituting (3.7′) into (3.6), we obtain



or in analytic form

$$\tilde{\tilde{A}}(p, p, q) = \frac{\Gamma_\alpha \Gamma_\alpha'}{\Pi(q^2) - q^2} \xrightarrow[q^2 \to 0]{} \frac{\chi a q_\alpha B_\alpha' + \chi' a q_\alpha B_\alpha + a^2 \chi\chi'}{q^2 \Pi(0)},$$

$$(\chi \equiv \chi(p, q), \quad B_\alpha \equiv B_\alpha(p, q), \quad \chi' \equiv \chi(p', q),$$

$$B_\alpha' \equiv B_\alpha(p', q)). \qquad (3.8)$$

In order to express $q_\alpha B_\alpha$ in terms of χ and a we make use of the transversality condition for Γ_α:

$$q_\alpha B_\alpha(p, q) = q_\alpha \Gamma_\alpha(p, q) - a\chi(p, q) = -a\chi(p, q). \qquad (3.9)$$

Substituting (3.9) into (3.8) and remembering that $\Pi(0) = a^2$ (cf. (3.4)), we find

$$\tilde{\tilde{A}}(p, p', q) \xrightarrow[q^2 \to 0]{} -\frac{\chi(p, q)\chi(p', q)}{q^2}. \qquad (3.10)$$

Comparing (3.10) and (3.5) we find that the pole at $q^2 \to 0$ has disappeared from $A = \tilde{A} = \tilde{\tilde{A}}$. Similarly all zeron and multi-zeron singularities disappear from any amplitude S on the mass shell.

If in some diagram $\tilde{S}$ for the amplitude S two points are joined by a zeron line, there will be another contribution $\tilde{\tilde{S}}$ to S in which the same two points are joined by a K*-meson line, the remainder of the diagram being the same as in S. Adding the diagrams $\tilde{S}$ with those of type $\tilde{\tilde{S}}$ we see that, similar to the situation in $A = \tilde{A} + \tilde{\tilde{A}}$, the singularity in S disappears at zero momentum of the zeron. (A detailed proof is not hard, but lengthy, therefore we omit it.)

As in any observable amplitude (A, J_α, etc.) the interference of zerons with K* mesons destroys all zeron poles and thresholds. This means that the zerons are fictitious particles: they can neither be emitted nor absorbed by real particles (otherwise the scattering amplitudes would exhibit zeronic singularities).

Nevertheless, a "shadow" of the zeron survives: off the mass shell all amplitudes (e.g., A(p, p′, q) in (2.3) or $J_\alpha(p, q)$ in (2.5)) have zeronic singularities, corresponding to a vanishing zeron mass. The fact that there are off-mass-shell singularities (poles or thresholds) not corresponding to physical particles is not a specific feature of the theory under consideration. If, for instance, the Green's function of the photon in ordinary quantum electrodynamics is chosen, such that its longitudinal part is singular, unphysical off-mass-shell singularities are found in all amplitudes. In distinction from quantum electrodynamics, in our case the singularities cannot be removed by changing the gauge of the K*-meson Green's function.

We note finally that we had no need to consider the initial interaction or the symmetry violation small. We have omitted only those diagrams which had no zeronic singularities and were thus inessential at $q^2 \to 0$.

4. POSSIBLE APPLICATIONS AND GENERALIZATIONS

The results derived in the preceding section can easily be separated from the specifically chosen model. In the case of spontaneous violation of an arbitrary symmetry group there will appear zerons with the quantum numbers of those group generators which are no longer conserved (cf. [5]). On the other hand, for generalized gauge invariance, the existence of a vector meson multiplet is necessary, having the quantum numbers of all the generators of the group.[8]

We have proved that through the interference of the vector meson with the zeron having the same quantum numbers, it destroys the latter and thus acquires a physical mass, in spite of the fact that its initial mass vanishes.

If the symmetry is not completely destroyed, there will be fewer zerons than there are vector mesons. Therefore, although all zerons disappear, not all vector mesons acquire a physical mass. (In the model under consideration the meson ρ^0_U remains massless, owing to strangeness conservation.) Therefore, if one considers the breakdown of SU(3) as a spontaneous effect,[5] one must admit that all the quantum numbers of this group are no longer conserved. In this case the electric charge should not be considered as directly related with these quantum numbers, i.e., there should exist supercharged particles.[10]

Another difficulty appears if one tries to identify the strangeness changing weak interaction currents with the corresponding currents associated with the generators of SU(3): the weak currents are approximately conserved and the currents of the group generators are exactly conserved, in spite of the mass differences.

In conclusion we note that under spontaneous symmetry breakdown, all the relations that follow from the current algebra[11] remain valid, since this latter method, consisting in the neglect of remote intermediate states, is related only to the smallness of the observed symmetry, but not with the mechanism which brings about this violation. Thus, in agreement with experiment, we are led to approximate universality of the vector meson interactions, the Gell-Mann–Okubo formula, etc.[4)]

We are grateful to V. G. Vaks and A. I. Larkin for useful advice and to K. A. Ter-Martirosyan for a discussion of the results.

APPENDIX A

THE ENERGETIC ADVANTAGES OF AN ASYMMETRIC VACUUM

We consider the energetic advantages of an asymmetric solution, assuming that it exists. We have seen in Sec. 2 that such a solution can be generated by an infinitesimal asymmetric perturbation of an initially symmetric Hamiltonian.

Let H_0 denote the symmetric Hamiltonian, and ϵV the symmetry breaking interaction; let E_0 be the ground state of H_0 and $E(\epsilon)$ the ground state of $H_0 + \epsilon V$. Energetic advantage means that $\lim_{\epsilon \to 0} E(\epsilon) < E_0$ for $\epsilon \to 0$. We prove that this condition is always fulfilled.

For this, we note that, owing to the variational principle,

$$E(\varepsilon) \leqslant (\Phi^* | H_0 + \varepsilon V | \Phi). \tag{A.1}$$

If one selects as a trial function Φ the eigenfunction Φ_0 of the ground state of H_0, one obtains

$$E(\varepsilon) < (\Phi_0^* | H_0 + \varepsilon V | \Phi_0) = E_0 + \varepsilon (\Phi_0^* | V | \Phi_0). \tag{A.2}$$

Since Φ_0 is symmetric and V violates the symmetry, $(\Phi_0^* | V | \Phi_0) = 0$. Consequently

$$\lim_{\varepsilon \to 0} E(\varepsilon) < E_0. \tag{A.3}$$

(This proof carries over automatically to statistical mechanics, if one makes use of the variational principle for the free energy.)

The model examples[2, 3] show what kind of contradictions appear when the symmetric solution, corresponding to an energetically disadvantageous vacuum, is used. It turns out that this "normal" solution, in distinction from the "superconducting" one, exhibits poles corresponding to "resonances" with negative lifetimes.

4)If the method used in Sec. 3 for the separation of the zeron pole is applied to γ_5-violation, one obtains in particular all the results usually derived from the hypothesis of a partially conserved axial vector current (PCAC). This is done in a paper by one of the authors [13].

APPENDIX B

STRANGENESS CONSERVATION IN A C-INVARIANT THEORY

In deriving the equation for $M_n(p) - M_\lambda(p)$ in Sec. 2, it was assumed that strangeness is conserved. At the same time we have seen in Appendix A that in order to assure the minimal energy for the vacuum, one must look for the solution exhibiting totally violated symmetry. We show, that if one assumes C-invariance of the solution, then it will automatically conserve strangeness. Indeed, let us not consider that strangeness is a priori conserved. Then the mass operator will involve transitions of the type $n \to \lambda$, $\lambda \to n$:

$$M(p) = \begin{pmatrix} M_{nn}(p) & M_{n\lambda}(p) \\ M_{\lambda n}(p) & M_{\lambda\lambda}(p) \end{pmatrix} \equiv M_0 + \mathbf{M}\boldsymbol{\tau}$$

$$\equiv {}^1\!/_2(M_{nn} + M_{\lambda\lambda}) + (M_{nn} - M_{\lambda\lambda})\tau_z + 2M_{n\lambda}\tau_x \quad \text{(B.1)}$$

(here τ_x and τ_z are Pauli matrices; τ_y does not occur in (B.1). Since, owing to C-invariance, M(p) is symmetric in isospin indices:[5] $M_{n\lambda}(p) = M_{\lambda n}(p)$).

In place of Eq. (2.2) for $M_z = M_{nn} - M_{\lambda\lambda}$ we will have an equation for $\mathbf{M} = (M_x, 0, M_z)$ in the form

$$\mathbf{M}(p) = \int d^4p' \mathscr{K}(p, p')\mathbf{M}(p'). \quad \text{(B.2)}$$

$\mathscr{K}(p, p')$ in turn, depends on isoscalars formed out of $\mathbf{M}(p)$:

$$\mathscr{K}(p, p') = \Phi[\mathbf{M}(p_1)\mathbf{M}(p_2)]. \quad \text{(B.3)}$$

In general, one could have added to the right-hand side of (B.2) an isovector of the form

$$\int d^4p_1 d^4p_2 \mathscr{L}(p_1, p_2, p)\mathbf{M}(p_1) \times \mathbf{M}(p_2), \quad \text{(B.4)}$$

but in our case, owing to C-invariance, $\mathbf{M} = (M_x, 0, M_z)$ is two-dimensional for all momenta p, and thus this term is missing.

Equation (B.2) is linear in $\mathbf{M}(p)$, therefore, in order for $\mathbf{M}(p) \neq 0$ (spontaneous symmetry violation), it is necessary that $\mathscr{K}(p, p')$ have unity as an eigenvalue. This eigenvalue can be expressed in terms of $\mathbf{M}(p)$ by means of Eq. (B.3). Then the condition for spontaneous symmetry breakdown is

$$\lambda[\mathbf{M}(p_1)\mathbf{M}(p_2)] = 1. \quad \text{(B.5)}$$

If the eigenvalue $\lambda = 1$ of the operator $\mathscr{K}(p, p')$ is nondegenerate,[6] it follows from (B.2) that

$$\mathbf{M}(p) = \mathbf{C}\chi(p), \quad \text{(B.6)}$$

where $\chi(p)$ is the eigenfunction of $\mathscr{K}(p, p')$, $\mathbf{C}$ is a constant vector, defined by the condition of spontaneous symmetry breakdown (B.5). But (B.5) defines only the length of $\mathbf{C}$, leaving the direction of $\mathbf{C}$ arbitrary. This is related to the fact that (B.2), (B.3) and (B.5) are invariant to rotations of $\mathbf{M}(p)$ by an angle which does not depend on the momentum p. Such a rotation implies a redefinition of the particles n and λ; by means of such a rotation one can achieve

$$\mathbf{C} = (0, 0, C_z), \quad M(p) = \begin{pmatrix} M_n(p) & 0 \\ 0 & M_\lambda(p) \end{pmatrix}. \quad \text{(B.7)}$$

Now n and λ do not transform into each other for any momenta, and thus strangeness is conserved.[7]

Both C-conservation and the nondegeneracy of the zeron were essential for our proof. Indeed, for an accidentally degenerate zeron we would have in place of (B.6):

$$\mathbf{M}(p) = \mathbf{C}_1\chi_1(p) + \mathbf{C}_2\chi_2(p) + \ldots \quad \text{(B.8)}$$

The solvability condition (B.5) would fix the angle between $\mathbf{C}_1$ and $\mathbf{C}_2$ and one could not direct both along the z axis. Thus, the diagonal character of $\mathbf{M}(p)$ for all p would not be possible and strangeness would not be conserved in virtual λ-n transitions. These transitions would lead to strangeness-violation in real processes of the type of $n + n \to \lambda + \lambda$ too. Without C-invariance we would also have an expression of the type (B.8) with all the consequences it implies.

Finally, we note that our assertion about the connection between conservation of charge parity and that of additive quantum numbers under spontaneous breakdown of SU(2) is confirmed by the concrete calculations of Arnowitt and Deser[12] in the electrodynamics of the muon and electron. In their approximate calculations both charge parity and the muonic charge are conserved, in analogy to strangeness in our example.

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[5]Here and below we call the U-spin isospin, for brevity. The isospin invariance of Eqs. (B.2) and (B.3) follows from the symmetry of the Lagrangian and can be easily verified on skeleton diagrams, similar to the derivation of Eq. (2.1")

[6]It can be seen from Sec. 2 that such a degeneracy means degeneracy of the zeron with respect to some quantum number foreign to the U-spin group. Therefore the degeneracy can only be accidental, and there is no symmetry from which it results. We also note that $\chi(p)$ in (B.6) is the $\lambda \to n$ + zeron (momentum q = 0) transition amplitude.

[7]We note that Eqs. (B.2) and (B.3) are not invariant under a rotation of $\mathbf{M}(p)$ by an angle which depends on p, therefore it is nontrivial that $\mathbf{M}(p)$ can be directed along the z axis for all p. (B.8) below is an example when this is impossible.

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