

# Simulating Human Visual Experience in Stadiums

Roland Hudson<sup>1</sup> and Michael Westlake<sup>2</sup>

<sup>1</sup>Universidad de Los Andes  
Bogota, Columbia  
r.hudson@uniandes.edu.co

<sup>2</sup>Populous  
London, UK  
Michael.Westlake@populous.com

## ABSTRACT

In this paper we describe progress to date of software that simulates occupant experience in high capacity sports venues. Our simulation aims to provide metrics that indicate quality of view, and in doing so generates data that indicates levels of human comfort. This approach enables the design process to be driven from the perspective of the occupant. In particular we implement a novel means of simulating and expressing quality-of-view that addresses deficiency's in the standard method of describing view quality. Visualisation of the simulation output is via an online 3D viewer shared with the entire design team. Views from any seat location can be inspected and data fields from the simulation can be compared. Data is represented with colour scales bound to a 3D seating bowl model. Using simulation to understand spectator experience from within a 3D environment challenges the validity of traditional design approaches that are based on two-dimensional thinking and drafting board logic. Our simulated study of view quality enables us to consider revisions to these traditional techniques which could lead to more spatially efficient seating facilities. Increasing spectator density is believed to enhance atmospheric qualities, this combined with better views will contribute towards an improved occupant experience.

## Author Keywords

View quality, stadiums, ergonomics, spectator experience

## INTRODUCTION

Simulation involves the creation of a model that attempts to allow the estimation of characteristics or behaviour of a system. The system that our software attempts to characterise is the organisation of spectator seating in a stadium, the characteristics of a specific seating configuration are measured in terms of view quality and physical comfort. Motivation for this work comes from a critique of the international standard for design and measurement of view quality in stadiums [1]. The current standard is inadequate because it uses 2D drafting logic rather than 3D computer graphic techniques, secondly it only considers view quality in a 2D domain, and lastly does not account for ergonomics. Our simulation operates within a 3D model space and applies computational techniques that incorporate concepts of human comfort in terms of range of view and body position.

In this paper we first provide a brief historical background and a review of the method given in current design standards. We then introduce our new simulation software, it's input requirements and the sequence of simulation. In order to provide a frame of reference for newer metrics our simulation is designed to evaluate view quality according to the existing standard. To achieve this, our software implements an inverse of the current methodology taking a given design as the starting point. To calculate a spectator's view quality using the current standard relative positions of neighbouring spectators are required. This need for context calls for a logic data structure of points representing the spectators. The series of algorithms that logically structure the data are discussed and then the method of calculation is described.

Next we discuss how the simulation captures three angles that suggest a range of comfort in movement of eyes, horizontal or vertical rotation of the head or even movement of the torso in order to perceive the full area of play. These simple angular metrics provide an immediate ergonomic aspect to the simulation.

We introduce a new method of measuring view-quality that forms a key component in our simulation. This technique applies computational geometry to capture a 3D view for every spectator in a stadium. The view is orientated and cropped in a way a way which relates directly to the position of the spectator.

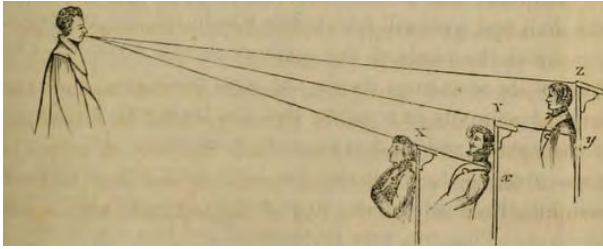
Given the quantity of data collected in the process of simulation we next describe how the numerical output is converted into a range of different coloured graphical forms included meshes and the export of data to a 3d web viewing environment. Last we discuss aspects of future work and design implications of our simulation.

## MOTIVATION

### History

John Scott Russell formalised what has become the standard methodology in 1838 with a published design method for auditoria [2]. Focusing on a stationary lecturer and direct lines of sight and sound, Russell identifies the need to progressively raise successive rows. The rate at which the row height changes is determined by a constant vertical offset from the top of seat back and a line drawn from the back of the seat to the speaker (figure 1). Russell describes a procedural drafting method. The result of this method is a set of points that describe as an equal-seeing or

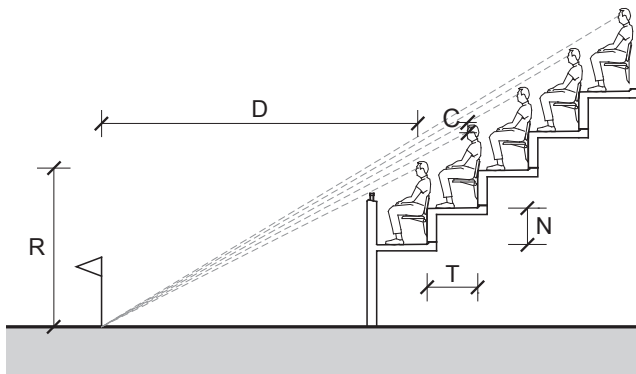
equal-hearing (iseidomal or isacoustic) curve. The technique was first applied by Adler and Sullivan in 1889 in designing the Auditorium Building in Chicago [3].



**Figure 1. Russell's iseidomal or isacoustic curve generating method. Resketched by authors.**

**The current c-value method.**

The current standard for stadium seating design is found in various statutory design guidelines for stadia [1, 4 and 5]. In these guides an equation is given to determine the vertical dimension C (the c-value) between the eye-point of one spectator and the sightline of the spectator behind. The focal point (the flag in figure 2) for field based sports is on the side line.



**Figure 2. C-value method.**

$$C = \frac{D(N + R)}{D + T} - R \quad (1)$$

Where:

D = horizontal distance from the eye-point to the focal-point

N = riser height

R = vertical height to the focal-point

T = seating row width

For 3D modelling the following form gives the spectator eye-points in the XZ plane assuming the focal-point is at the model origin.

$$x_r = x_{r-1} + rw \quad (2)$$

$$z_r = z_{r-1} + c + \frac{rw(z_{r-1} + c)}{x_{r-1}} \quad (3)$$

Where:

$r$  = row number

$rw$  = row width

$c$  = c - value

Initial horizontal and vertical dimensions between the first eye-point and the origin are required to locate the first spectator and initialise the series, and the equations are usually extended (in the context of football) to allow for the view of the first spectator over an advertising board between the spectators and the playing area. Using vertical (height of a seated spectator) and horizontal offsets (distance to the front of the step) the vertices for a stepped section line can be generated. To define the three-dimensional seating bowl surface the section is swept around the stadium. The c-value is used to define the generating section and to refer to the quality of view for the entire stadium – a higher value indicating a better view.

**Critique**

Little has changed between Russell's method of 1838 and that used to design international stadiums today. Although it has been in use for almost 180 years, it has not received a critical review or detailed empirical study despite changes in the design technology available and understanding of human vision. The primary critique of the method in current use is that it is 2D and based on a system where the objective was to ensure sight and acoustic lines to a stationary point. In this 2D system views are considered to be perpendicular to a spectator's shoulders and to always pass above the heads of spectators-in-front. In fact views may be between the heads of those seated on the row in front and spectators move their eyes, head and shoulders during a game. The c-value method also does not account for height above the playing surface which provides a less obtuse viewing angle of the pitch plane. The c-value method uses a static focal-point located at the edge of the playing area, the focus of play in sports events rapidly changes and ranges all over the playing area.

Others have noted the deficiencies in the method and proposed a more rigorous computational approach [6]. In a low capacity context (theatres with less than a few thousand seats) a more robust approach has been investigated along with development of a series of metrics that relate to visual comfort [7] this has not yet been extended to sports facilities.

**MODEL SET UP AND POINT SORTING ALGORITHMS**

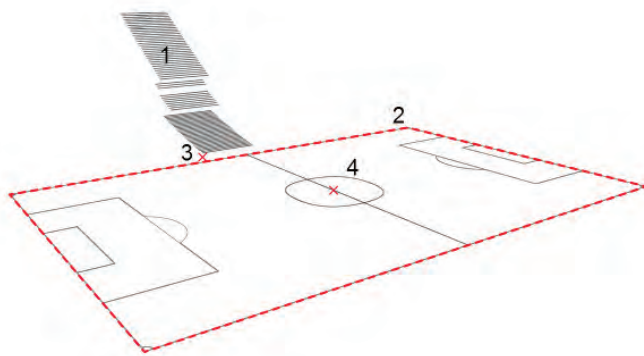
**Basic model inputs and over view of simulation sequence**

In order to directly address the shortcomings of the traditional method our simulation requires a 3D environment where every spectator eye-point is represented. The entire playing area is defined along with a point that represents the centre of visual attention for all spectators. A reference point is required to structure the

eye-points in a way that is analogous to the physical seating layout.

Our simulation is a plug-in for the Rhino3d modelling application [8]. The plug-in is written with C# and uses the rhinocommon software developer kit [9]. All inputs and output are accessed through the plugin's graphical user interface. To initiate a simulation four geometric inputs (figure 3) are required:

1. Eye-points - a set of points representing spectator eyes.
2. Pitch-boundary - a closed planar curve that represents the edge of the playing area.
3. Sorting reference point - a single point located closest to the eye-point in the first row.
4. Centre of visual attention - a single point used for view orientation. Typically the pitch centre point.



**Figure 3. Geometric inputs for simulation.**

The eye-points can be passed in as an unstructured set, the simulation defines the structure of the point data. The pitch-boundary does not need to be rectilinear, any closed curve can be used allowing simulation of baseball or cricket venues. Once the points are selected they are organised into a data structure using the reference point. The user then selects the type of analysis results required and the simulation steps through every eye-point and simulates and stores metrics for each.

#### INVERTING THE C-VALUE METHOD

Our software enables analysis of view-quality in stadiums using geometry that describes an existing or proposed 3D environment. We implement new methodologies that indicate the spectators experience in terms of ergonomics and the amount of the playing area visible. In order to compare these metrics with the traditional means of describing quality of view we provide the ability to analyse the stadium in terms of the c-value.

Using equations (2) and (3) we can determine c-value for any eye position in the XY plane.

This can be further generalised to work in any plane if the values for z and x are replaced with horizontal and vertical dimensions between a focal-point with any coordinates.

$$c = (\Delta x_r - rw) \left( \frac{\Delta z_r}{\Delta x_r} \right) - \Delta z_r - (\Delta z_r - \Delta z_{r-1}) \quad (5)$$

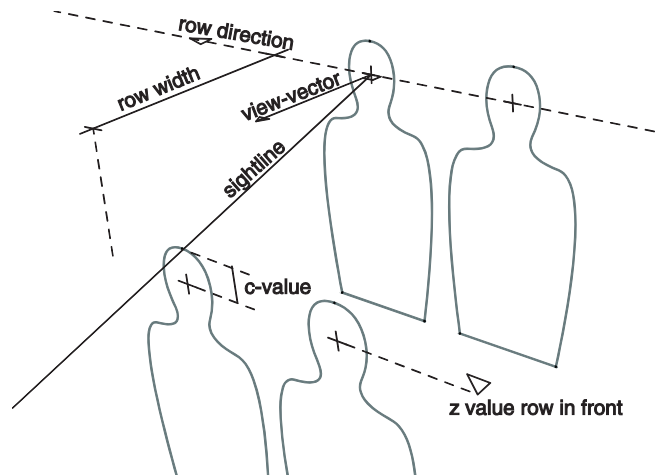
Where:

$$\Delta x_r = x_r - fp_{x_r}$$

$$\Delta z_r = z_r - fp_{z_r}$$

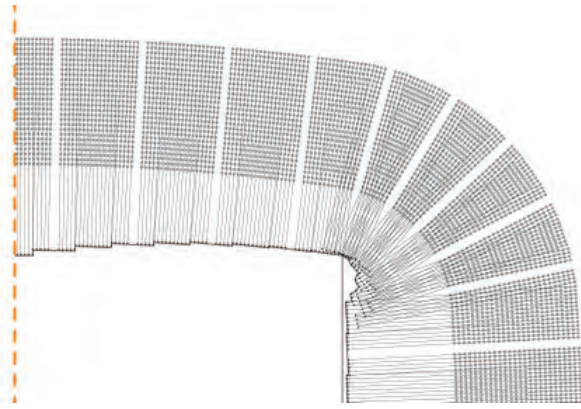
$$fp = focalpoint$$

Calculation of the c-value is therefore context dependent. For every eye-point three items are required: the level of the row in front, the focal-point and the row direction. To find the focal-point we first define a view-vector which is perpendicular to the row direction (figure 4). The focal-point is found using one of three methods that the user specifies, closest-point, perpendicular and through-corner.



**Figure 4. Context required to inversely calculate c-value.**

The closest-point method finds the focal-point by evaluating the pitch-boundary and finding the closest point. The perpendicular method generates a ray for each of the pitch-boundary edges, each is intersected with the viewing normal and the intersection closest to the spectator eye-point is the focal-point. For the through-corner method (figure 5) the closest vertex on the pitch-boundary polygon is found and a ray parallel to the seating row is defined at this point. The focal-point is the intersection between this ray and the viewing normal.



**Figure 5. Positions of c-value focal points.**

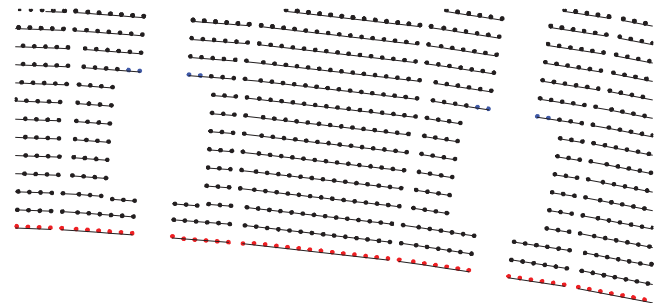
### Eye-point sorting algorithm

The need for context determines a three stage point sorting system that organises eye-points into a data structure analogous to a physical row and seat numbering system. First eye-points are sorted by absolute level, second into a sequence along the row and third into groups along the row that lie on a common straight line.

Eye-points with the same z-coordinate (within a user defined tolerance) are considered on the same row and share a common array index. Each row of eye-points is next sorted into a sequential order. This is calculated by determining the angle at the centre of the pitch-boundary between the sorting reference point and the spectator point. The points are sorted by angle size with the smallest first. Along each row the eye-points are grouped according to which ones share a common straight line (which corresponds to the structural elements used to construct the seating bowl (referred to as the riser)). The riser determines the row direction and perpendicular to this is the spectator's view-direction. The angle at each eye-point is measured between its two neighbours. If the angle between these 3 points is not 180 degrees (within a user defined tolerance) a new riser group is defined (the second dimension of the data structure). The data structure has three dimensions, the last dimension is determined by the sequential position of the eye-point along the riser.

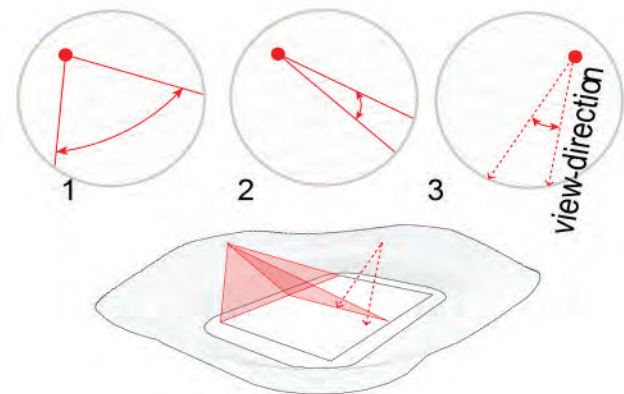
The need for context to calculate the c-value is a further deficiency in the c-value method. Where c-value cannot be calculated view quality for these locations cannot be expressed. During analysis the eye-points are classed according to their physical context e, four conditions exist; front-row, open-row, single-seat or full-context. C-value cannot be calculated for the front-row of any tier – since a row in front is always required. An open-row situation is one where there is no row directly in front of the eye point but it is not a front row, for example the row directly above a vomitory opening. Single-seat is the situation where only one seat exists on the riser, here it is not possible to calculate the viewing direction and therefore the c-value focal-point cannot be correctly found and no c-value can be calculated. Full-context is where the c-value can be

correctly calculated according the published method. Figure 6 shows point data structure and cases where insufficient data exists to calculate c-value.



**Figure 6. Data structure for spectators around two vomitories each line represents one riser. Colour of circles indicates the amount of context. Red = front-row, blue = open-row, black = full-context.**

Structuring of data in a meaningful manner provides the context that allows calculation of the c-value. This data structure also enables the simulation to include a seat-position-navigator interface that allows the user to set the model view to a specific seat on a specific row and visually assess the view.



**Figure 7. Viewing angles.**

### HORIZONTAL AND VERTICAL VIEW ANGLES

Viewing angles indicate spectator comfort in terms of how far a spectator will need to turn their head or body. Our simulation provides three angular measures (figure 7):

1. Horizontal view angle - angle measured in a horizontal plane at the eye-point to the extreme left hand and right hand points on the projected playing area.
2. Vertical view angle - angle measured in a vertical plane at the eye-point to the extreme upper and lower points on the projected playing area
3. Torsion angle - angle measured in a horizontal plane at the eye-point between the view-direction (perpendicular to row) and the centre of visual attention, represents the movement of the eyes (or turning of head or torso in extreme cases).

### ALGORITHM FOR CALCULATING A-VALUE

A-value is an innovative method that aims to quantify view quality and address the shortcomings of the c-value method discussed above. A-value measures the area of the playing surface that is projected into the spectators view plane, expressed as a percentage of the total area visible. Firstly, unlike the c-value it takes into account the entire playing area and not just a stationary point. For each spectator the view of the pitch-boundary or playing area is simulated, this view includes other spectators and elements from the building and aspects of human vision.

#### Basic a-value simulation

The basic simulation of the a-value is the calculation of the area of the polygon found when the pitch-boundary is projected onto the spectator's view-plane. The vector between the spectator's eye-point and the centre of visual attention defines the normal vector of the view-plane. This view-plane is assumed to be orientated to the spectator's centre of attention. The view-plane can be located at any point along the view-vector other than at the eye-point. Between each vertex of the pitch-boundary and the eye-point a vector is defined, each vector is intersected with the view-plane. The closed polygon defined by the intersection points is the projected pitch-boundary. Using a basic view frustum with a field of view of 60 degrees and orientated using the view-vector and a global z-vector a clipping rectangle or basic view-boundary is created on the view-plane. The projected pitch-boundary is clipped with the view-boundary using the Sutherland-Hodgman (S-H) algorithm [10]. The a-value is the area of the resulting clipped polygon expressed as a percentage of the total area of the view-boundary (figure 8).

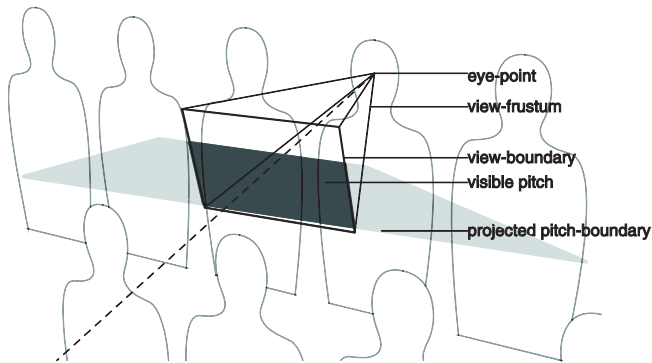


Figure 8. Basic A-value simulation.

The simulation sequence described is, in fact a simple process that is used to determine how to present any 3D geometry onto a screen. The innovation in this paper is in the application of this to simulate quality of view by tuning and considering the frustum as a simulation of human vision and secondly the analysis of the resulting projected geometry and storage of the result for later reference. Within the simulation geometric details relate the a-value directly to the human body. Using the view-vector to define the view-plane means that the result is directly related to the

where the spectator's visual attention is directed. The field-of-view of the frustum can be considered to be a basic representation of the human cone of vision [6], the area of the projected pitch-boundary outside of this ignored.

### Results and comparison

The quality that the A-value represents is the projected area of pitch, therefore seats in upper levels generally have higher A Values (figures 9+10). After a certain distance from the pitch the A-value can be seen to diminish, this can be seen on the upper rows in figure 10. Analysing C Value throughout a stadium shows higher values in lower tiers, constant values along the generative section and diminishing values with distance from the pitch. With the exception of the front rows in each tier and boxes diminishing values can be seen in figure 11. In general terms we can observe that higher A Values indicate a better overall view of the pitch while higher C Values indicate proximity to the action.

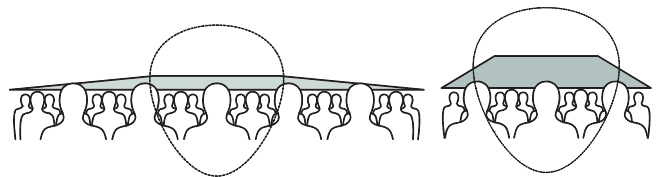


Figure 9. Difference between projected pitch area in a lower row (left) and upper row (right).



Figure 10. A-value results, higher values in upper rows.

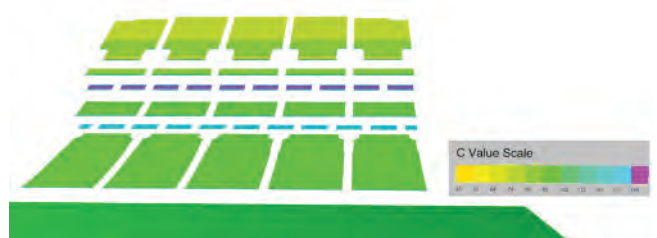


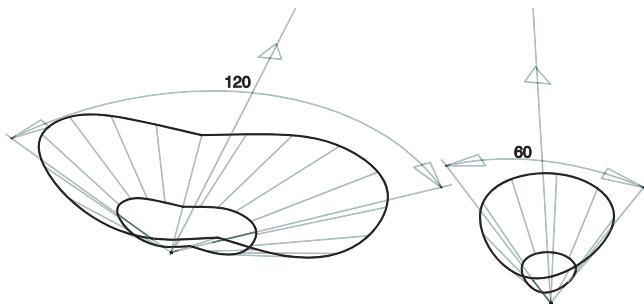
Figure 11. C Value results, higher values in lower rows

#### Simulating human cones of vision

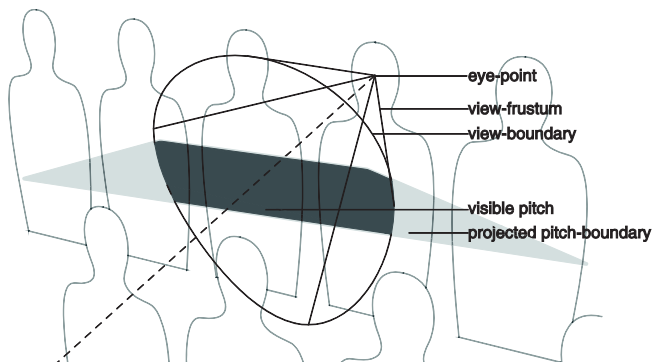
The basic frustum is a crude simplification of the human field-of-view. The simulation can be configured to include ergonomically defined viewing cones that have been used in other fields [6]. The simulation enables the a-value to be calculated as that area which lies within the effective binocular field-of-view (as defined for driving regulations in the UK [11]). The form of the binocular field-of-view reflects the shape of the human head and specifically the position of the eyes in the skull. In particular the frontal

bone above the eye restricts the view above while the lower zygomatic and maxillary bones are more recessed [12] permitting a wider angle of view.

Two fields-of-view can be simulated, a stationary head position and a dynamic cone-of-vision based that includes head rotation. 30 degrees of rotation is considered an unstrained rotation for a human [6 and 13]. The static binocular field-of-view is swept 30 degrees either side of the primary view direction to give a dynamic binocular field-of-view (figure 12). The result of this is a redefinition of the view frustum from a basic truncated pyramid to a more complex cone like form. The view-boundary is now defined by the intersection of the view-plane and one of these cones. Substituting one of these view-boundaries into the basic a-value calculation procedure provides a modified a-value that now comes closer to representing the projected-pitch area as a proportion of a more accurate human cone of vision (figure 13).



**Figure 12. Human cone of vision frustums Left: Static binocular field of view. Right: Dynamic binocular field of view.**



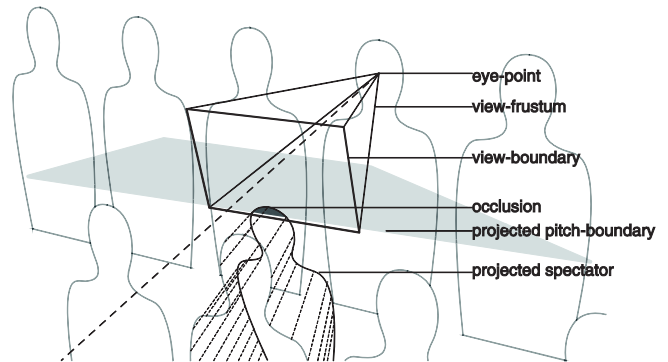
**Figure 13. Simulating a-value with the static binocular field of view.**

### View Occlusion

The form and area of the visible projected pitch-boundary give an initial indication of view quality. To move towards a more detailed understanding of the relationship between the form of the seating bowl and quality of view from any seat we must consider the spectators sitting directly in front. The amount that these spectators can block the visible pitch-boundary depends on the difference in vertical height and the relative horizontal position between seats on two successive rows. If the vertical difference is great enough or

if the view-vector passes between two heads the view may not be significantly occluded. The c-value method could not account for the second of these scenarios. Our simulation can efficiently calculate the proportional area of pitch-boundary that is blocked by the heads of spectators in front.

Given the visible, projected pitch-boundary the next step in pursuing a view with verisimilitude is to include outlines of the spectator's in seats in front. To maximise efficiency of this calculation we make two assumptions. First that the spectator in front can be represented with a planar polygonal outline orientated (see those in figure 14) to the seating row. Secondly that only those spectators in the four rows in front may have an impact. A standardised two-dimensional head-and-torso outline (of a seated human form seen from the back) is stored as of a part of the simulation software.



**Figure 14. Simulating occlusion.**

To begin to simulate the view occlusion a sub-set of eye-points is defined that includes only those that lie within the selected view-cone and are located on the four rows in front. For each of these eye-points the orientation of the seat is needed to transform the predefined head-and-torso outline. The structuring of the original eye-point data set provides the necessary context to determine the direction of any seating row. Using the row direction and the global z-vector a plane is defined. The vertices of the head-and-torso outline polygon are transformed to this plane and then projected to the view-plane (figure 14). Using the S-H algorithm the head-and-torso outline can be clipped with the already clipped, projected pitch-boundary. Once clipped, the area of the head-and-torso outline is stored and the process is repeated for each of the points in the set. Each time, the area of the clipped, head-and-torso outline is added to the cumulative total until eventually the total occlusion area is expressed as a percentage of the proportion of the total view-boundary.

### View Obstruction using Image Segmentation

We distinguish between occlusion and obstruction. Occlusion (described above) in our simulation refers to the area of projected pitch-boundary blocked by other spectators and is a formal component in simulating a-value. The term obstruction is used to refer to all other objects that can disrupt the spectator's view of the event. Our current

simulation technique calculates obstruction using an image segmentation method that captures rendered views and analyses them to determine the quantity of pixels of certain colours. (This method was in fact first implemented to calculate occlusion but found to be much slower than the current projective method). We render the pitch as green and the potential obstruction objects as red our simulation returns the percentage of each as a proportion of the total viewing-boundary area. The orientation for the camera of the rendered view is defined using the same geometry as the occlusion method. The camera position is set at the eye-point, the global z-vector defines the up-vector for the camera and the camera-target is the centre-of visual-attention. The captured bitmap can be masked with a form defined using one of the two ergonomic view-cones.

Differentiating between occlusion and obstruction provides the option of controlling what elements are being tested and can therefore be used in a variety of ways. Two examples of situations that have benefited from this approach are evaluating the impact of adding a horizontal advertising to an existing stadium and testing the intrusion of architectural metal work such as handrails. In both cases the geometry of interest is modelled and included in the model environment. For each eye-point the simulation returns the area of pitch visible with and without the test geometry and the total area of test geometry within the spectators view.

This obstruction simulation methodology serves the purpose of proof of concept and in future versions of our simulation software we intend to implement more sophisticated frustum culling methods. The objective is to efficiently evaluate complex scenarios that include all architectural geometry and objects within a contemporary sports stadium.

Implementing an image analysis method as part of the simulation allows views to be saved for later use. In some competitive processes for stadium bids this is a basic requirement. As the simulation generates the images each one is named and referenced to a physical location inside the stadium using the indexing generated when structuring the data. The images can be accessed using an html interface that allows specific views to be accessed via a web interface.

### VISUALISATION OF RESULTS

Views from 50000 seats can be simulated in about 90 seconds, 10 different metrics are generated for each seat giving half a million data items. Processing and visualisation of results therefore forms an important component in the software in order to enable the comparison of several alternative designs. Any one of the metrics can be selected to generate a coloured graphical output that is placed on its own layer in the original rhino model. The colour scales and their numerical ranges are customisable, start and end colours can be selected and the number of divisions in the scale defined. To preserve saturation when interpolating linearly between colours HSL

colour coordinates are used. Possible graphical outputs are coloured points, squares or circles, one for each eye-point. Using a Delaunay triangulation algorithm the eye-points can be used to generate a mesh (figure 15). The selected metric determines the colours that are assigned to the mesh vertices. Larger mesh triangles are removed to leave a mesh that represents the surface of the seating bowl. The mesh can be exported to a JSON file and shared using an online 3D viewer developed with a JavaScript and Three.js [14] a library enabling simplified implementation of WebGL animations.

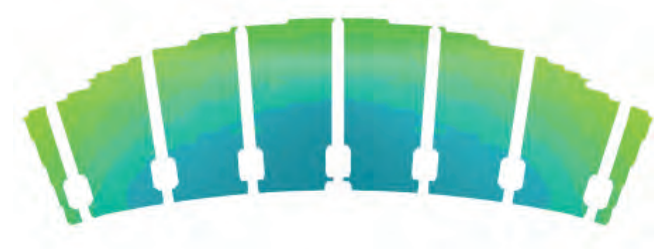


Figure 15. Coloured 3D mesh output.

In addition to coloured graphical output any metric can be recorded in the model alongside its corresponding eye-point as a number in text. Viewing angles can be shown as a small triangle placed in the corresponding plane at the eye-point. Any data can be written directly to excel for further statistical analysis, we have also implemented a custom file format where the model structure and all simulation results can be stored and reloaded at any time.

### SUMMARY

At the core of our simulation of spectator experience is a novel method of conceptualising view quality. A-value or area-value is a measure of view quality that uses the area of the polygon found when the pitch-boundary is projected into the spectator's viewing-plane. By projecting outlines of spectators in front to the view plane we simulate the amount of occluded view and by including static architectural elements within the simulation environment we determine the amount of obstructed view. We extend this view quality concept further and the simulation includes recognized norms for human cones of comfortable vision when the head is static and with a small rotation about the neck axis. View cones, projected pitch-boundary and the heads-of-spectators-in-front are all represented as 2D polygons in a view plane, to combine these and derive A-value for the a-value we implement the Sutherland-Hodgman convex polygon clipping algorithm in an iterative fashion.

The a-value captures a series of aspects of view-quality that were previously inaccessible using the standard c-value methodology. This new simulation process implements basic computational geometry methods in contrast to the drafting techniques of the c-value method. Through the simulation of viewing angles and the construction of spectator views that are clipped to ergonomically defined

fields-of-vision our software provides insight into human comfort.

#### FURTHER WORK

The use of the A-value in the design of sports venues is at an early stage. We have applied this work directly to the development of three international stadiums. Our applications to date have involved the comparison of alternative designs. We believe however that the use of A-value and other metrics can be incorporated in generative methods that ensure certain minimum standards are met. In moving towards a generative mode our immediate work includes detailed study of the ergonomic role the view cones. We plan a series of detailed studies of typical existing stadiums and detailed analysis of the results to determine which metrics are the most significant and where correlations if any lie.

Our simulation has been developed with a focus on soccer and rugby stadiums, it is immediately applicable to most field sports and indoor stage based venues. The software generates indicators that can be used to evaluate view quality where attention is focused on a vertical or horizontal plane. Further research is required to extend applicability to track venues.

Spectator experience is not only limited to in-seat comfort and access to a clear field-of-view. We have identified a series of additional metrics which when implemented will add further dimensions to the ability to predict the quality of experience in stadiums.

*Seat integration* would describe of connectivity of a spectator to the stadium facilities. This metric would draw on the work of Space Syntax [15] and apply the concept of universal distance and analysis of spatial integration.

*Pitch targeting.* Contemporary sports analysis commonly involves the capture of data describing playing area usage through sensors attached to players and balls. These data sets could be used to identify and specify particular areas of playing field that each spectator must have a certain view of.

*Social recognition metrics.* The capacity to recognise fellow occupant's faces and see facial expressions is particularly important in some venues (theatres, meeting rooms and lecture halls). Measuring the number of faces that lie within range of a spectator's visual acuity would provide an indication of how well socially a space is configured.

#### ACKNOWLEDGMENTS

Populous.

#### REFERENCES

1. HMSO. *Guide to safety at sports grounds*. Fifth Edition. (2008). 109.
2. Russell, J,S. Elementary Considerations of some principles in the Construction of buildings designed to accommodate spectators and auditors. *Edinburgh new philosophical journal*, 27, (1938), 131-136.
3. Forsyth, M. *Buildings for Music*. The MIT Press, Cambridge, MA, USA. 1985.
4. John, G., Campbell, K. *Handbook of Sports and Recreational Building Design*. Second Edition. The Architectural Press, London, UK, 1995.
5. John, G., Sheard, R., Vickery, B. *Stadia, Fourth Edition: A Design and Development Guide*. Taylor & Francis, London, UK, 2007
6. Ham, R. *Theatres, planning guidance for design and adaptation*. The Architectural Press, London, UK. 1972.
7. Dritsas, S., Rafailaki, E. A computational framework for theatre design. *Proc Embodying virtual architecture: The third international conference of ASCAD*. (2007), 165–182.
8. Rhinoceros, (2014, November 13). Retrieved from <http://www.rhino3d.com/>
9. RhinoCommon Plug-in SDK, (2014, November 13). Retrieved from <http://wiki.mcneel.com/developer/rhinocommon>
10. Sutherland, I., Hodgman, G.W. Reentrant Polygon Clipping. *Communications of the ACM*, 17, (1974), 32–42.
11. Nixdorf, S. Physiology of viewing. In *Stadium Atlas: Technical Recommendations for Grandstands in Modern Stadia*, edited by Stephen Nixdorf, Wiley, London, UK. 2008, 130-139.
12. Hogan, M.J., Alvarado, J.A., Weddell, J.E. *Histology of the Human Eye*, Saunders Elsevier, St. Louis, MO, USA, 1971.
13. Karhu, O., Kansu, P., Kuorinka, I.. Correcting working postures in industry: A practical method for analysis. *Applied ergonomics*, (1977), 8, 4, 199-201.
14. Three.js (2014, November 13). Retrieved from <http://threejs.org/>
15. Hillier, B. *Space is the Machine*. The Press Syndicate of the University of Cambridge, Cambridge, UK. 1999.